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Coherent Risk Measures

HSBC Quant Academy for AGH WMS

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Table of Contents

Contents

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1 Introduction: What is Coherence and Risk?

Notice: Questions Ahead

Question: How are you today?

(A) Excellent

(B) Good

(C) Not so good

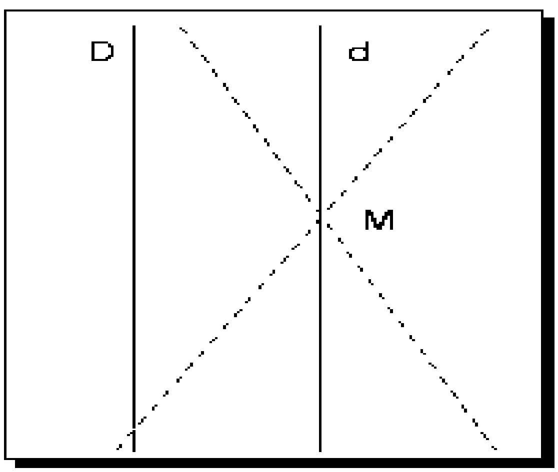

Figure 1: Euclid Proposed 5 Axioms (or rather 3 + 2 definitions) in his "Elements" as foundation of Geometry. — see eg. Heath, 1909

Are other paradigms possible

Question: Other paradigms - sets of axioms?

Can it be coherent that more than one line through a point (not on line D) never intersects with line D

(A) No

(B) Yes, but only a finite number of lines

(C) Yes, it is possible that an infinite amount of lines through that point that will never cross D

Alternative Coherent Geometries

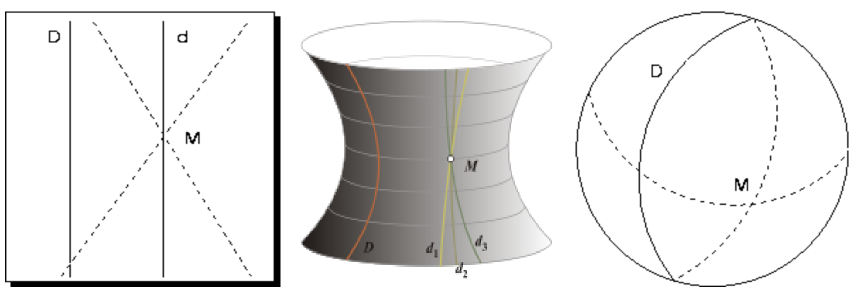

Figure 2: Alternative coherent geometries. Where in Ecuclid's geometry there is exactly one line parallel to line D and through point M, in Nikolaï Lobatchevski's hypersphere there are an infinite number and in Bernhard Riemann's sphere there are none.

Thinking about Financial Risk

Definitions of Risk Measures

Definition 1 (Standard Deviation / Variance)**.**

$$VAR := \textbf{variance} = E[(X - E[X])^2]$$

$$\sigma := \text{standard deviation} = \sqrt{VAR}$$

Idea	Reference
no risk, no rewards	Ecclesiastes 11:1–6 (ca. 300 BCE)
diversify investment	Ecclesiastes 11:1–2 (ca. 300 BCE) and Bernoulli, 1738

Table 1: Key ideas about investment risk

Risk Measure	Reference
variance (**VAR**)	Fisher, 1906, Marschak, 1938 and Markowitz, 1952
Value at Risk (**VaR**)	Roy, 1952
semi-variance (**S**)	Markowitz, 1991
Expected Shortfall (**ES**)	Acerbi and Tasche, 2002 and De Brouwer, 2012

Table 2: Normative theories and their risk measures implied.

Definition 2 (Value-at-Risk)**.**

$$VaR_\alpha(\mathcal{P}) := -(\text{the best of the } 100\alpha\% \text{ worst outcomes of } \mathcal{P})$$

Definition 3 (Expected Shortfall)**.**

$$ES_{(\alpha)}(\mathcal{P}) := -(\text{average of the worst } 100\alpha\% \text{ realizations})$$

Definition 4 (Worst Expected Loss)**.**

$$WEL := \textbf{Worst Expected Loss} = -E[\min(\mathcal{P})]$$

What Risk Measure is your Favourite?

Question: Best risk measure for banking

According to you, what risk measures should banks use?

(A) standard deviation / variance (σ / VAR)

(B) value at risk (VaR)

(C) expected shortfall (ES)

(D) worst expected loss (WEL)

Visualization of some risk measures

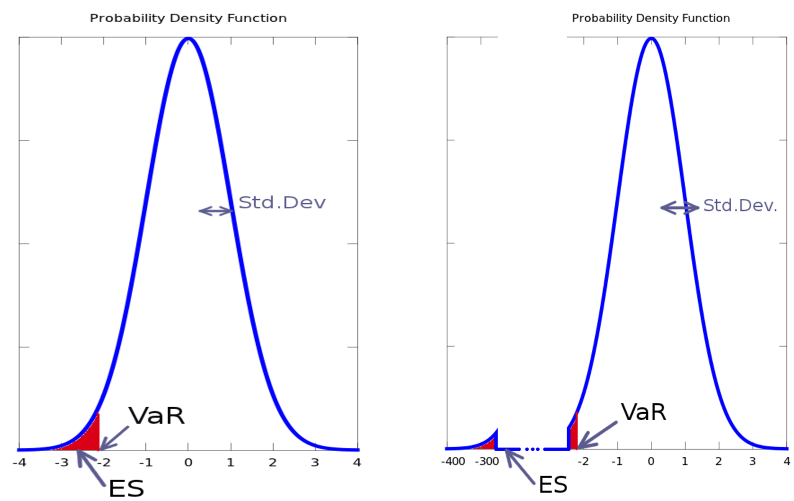

Figure 3: visualization of ES, VaR and σ. Note that WEL is not defined.

2 An Axiomatic Approach to Financial Risk

2.1 Axioms

A set of Axioms
Proposed by Artzner et al., 1997

Definition 5 (Coherent Risk Measure)**.** A function $\rho : \mathbb{V} \mapsto \mathbb{R}$ is called a **coherent risk measure** if and only if

A. **monotonous**: $\forall X, Y \in \mathbb{V} : X \leq Y \Rightarrow \rho(X) \geq \rho(Y)$

B. **sub-additive**: $\forall X, Y, X+Y \in \mathbb{V} : \rho(X+Y) \leq \rho(X) + \rho(Y)$

C. **positively homogeneous**: $\forall a > 0 \,\text{and}\, \forall X, aX \in \mathbb{V} : \rho(aX) = a\rho(X)$

D. **translation invariant**: $\forall a > 0 \text{ and} \forall X \in \mathbb{V} : \rho(X+a) = \rho(X) - a$

Law-invariance under P: $\forall X, Y \in \mathbb{V}$ *and* $\forall t \in \mathbb{R} : P[X \leq t] = P[Y \leq t] \Rightarrow \rho(X) = \rho(Y)$

Which Risk Measure is Coherent?

- **VAR (or volatility)** is not coherent because it is not monotonous (trivial)
- **VaR** is not coherent, because it is not sub-additive Artzner et al., 1999
- **ES** is coherent Pflug, 2000
- **WEL** is not usable because it is not Law-Invariant

. . . but who should care?

2.2 Spectral Risk Measures

Spectral Risk Measures

Definition 6 (Spectral Risk Measure)**.** Let X be a stochastic variable, representing the return of a financial asset. Then we define the **spectral measure of risk** $M_\phi(X)$ with **spectrum (or risk aversion function)** $\phi(p) : [0,1] \mapsto \mathbb{R}$ as:

$$M_\phi(X) := -\int_0^1 \phi(p) F_X^{\leftarrow}(p)\,\mathrm{d}p$$

Coherence for Spectral Risk Measures

Theorem 7. *The risk measure $M_\phi(X)$ as defined above is coherent, if and only if*

$$\begin{cases} \phi(p) \text{ is positive} \\ \phi(p) \text{ is not increasing} \\ \int_0^1 \phi(p)\,\mathrm{d}p = 1 \end{cases}$$

Proof. See Acerbi, 2002 □

This theorem proves that there is a deep relation between what we have defined as "coherent" and a non-increasing risk spectrum. A not-increasing risk spectrum means that in calculating the risk measure, one cannot assign a lower weight to a worse outcome. In other words the spectrum $\phi(p)$ of the risk measure M_ϕ determines the weights associated to possible outcomes. This explains the alternative name for $\phi(p)$: "risk aversion function".

A person who thinks coherently cannot allocate a higher weight to better outcomes in a risk measure, and hence the risk aversions function is not increasing. And this is what VaR does: it assigns a zero weight to all outcomes worse than a certain quantile: in Equation 2 one can see that the spectrum of VaR increases infinitely steep just before α.

The Spectrum of ES

Example 8. The spectrum or risk aversion function for the α-Expected Shortfall (ES_α) is

$$\phi_{ES_\alpha}(p) = \frac{1}{\alpha}\mathbf{1}_{[p\leq\alpha]} := \begin{cases} \frac{1}{\alpha} & \text{if } p \leq \alpha \\ 0 & \text{else} \end{cases} \tag{1}$$

The Spectrum of VaR

Example 9. The spectrum or risk aversion function for the α-VaR is the Dirac delta function:

$$\phi_{VaR_\alpha}(p) = \delta(p-\alpha) \tag{2}$$

The spectral representation of risk measures clarifies a lot.[1] Via this presentation it is possible to determine necessary and sufficient conditions on the spectrum for a risk measure to be coherent.

3 Case Studies

3.1 Default Risk of Bonds

Case 0

One Bond

Question: One Bond

Assume one bond with a 0.7% probability to default in one year in all other cases it pays 105% in one year. What is the 1%VaR?

(A) -\$5 (for a nominal of \$100, it is -\$5)

(B) \$0

(C) 0.7% (for a nominal of \$100, it is \$0.7)

(D) something else

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[1] More details can for example be found in De Brouwer (2012), Acerbi, 2004 and most of the proofs are in Acerbi (2002).

Note: continuity in α

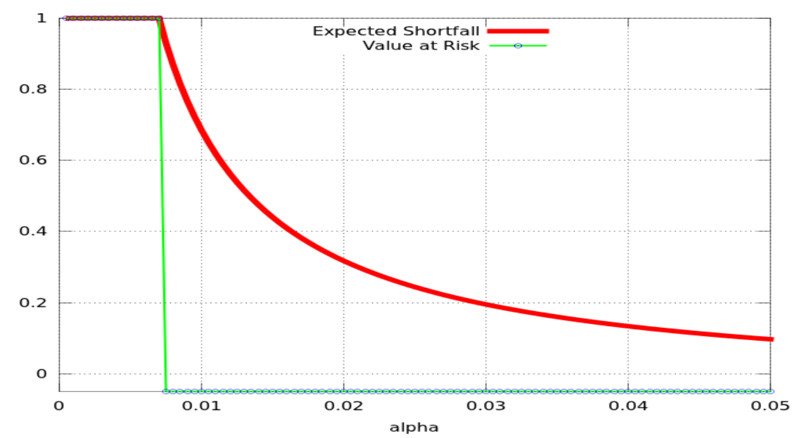

Figure 4: ES and VaR in function of α for one bond.

In the case of one bond that has a $\alpha\%$ probability to default and in all other cases pays back the nominal, we note that the VaR does not see any risk as long the α is below that probability.

Case 1

Question: Two Independent Bonds

Consider two identical bonds with the same parameters, but independently distributed. What is the $1\% VaR$ now?

(A) -5%

(B) 0%

(C) 0.49%

(D) 47.5%

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The Cumulative Distribution Function

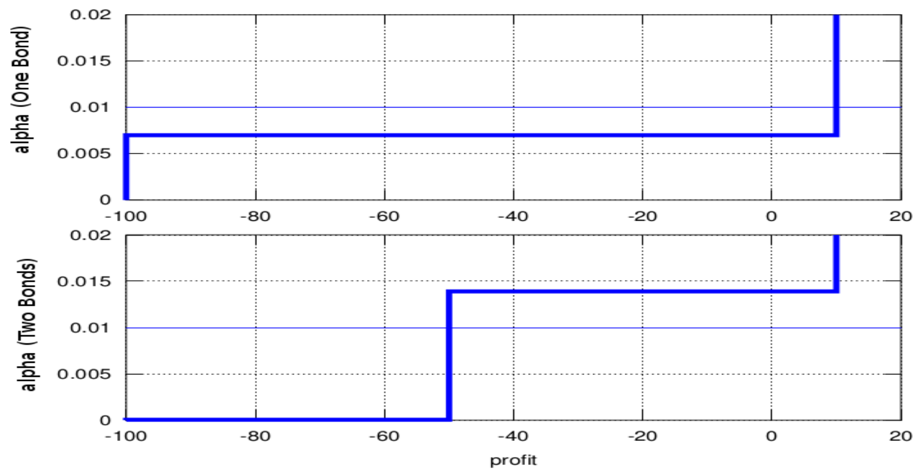

Figure 5: The cdf of $\mathcal{P}$ for one and two independent bonds.

Case 2
The Evil Banker and his customers

Question: The Evil Banker's First Dilemma

Consider an Evil Banker who has to compose a portfolio for his private client. If there is at least one default in the portfolio, then the banker will loose that client.
How can our banker minimize his work and maximize his income?

(A) propose a well diversified portfolio and explain the importance of diversification

(B) propose to invest everything in one bond

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Case 3

Question: The Evil Banker's Second Dilemma

Consider an Evil Banker who has to comply with Basel III, hence uses for assessing market risk VaR. Being Evil he does not care about the size of a bailout. So how does he minimize VaR?

(A) Invest in a well diversified porftolio

(B) Invest everything in one bond

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3.2 More Bonds

Case 5
More Bonds

Example 10 (N Independent Bonds). Consider now an increasing number of independent bonds with the same parameters as in previous example.Trace the risk surface.

Risk in Function of Diversification
Convecity (I)

The Risk Surface
Convexity (II)

3.3 Risk Reward for Gaussian Assets

The Three Simple Assets Classes used below The following hypothetical assets (or asset classes) are used in many basic examples.

A. a volatile asset class, e.g. "a well-diversified portfolio of equities, such as a world equity fund or funds".

B. a moderately volatile asset class: this could be understood as "a well-diversified portfolio of bonds such as bond investment funds",

C. a safe asset class with low variance: this could be taken to be "a well-diversified portfolio of cash and near-cash holdings, such as cash investment funds",

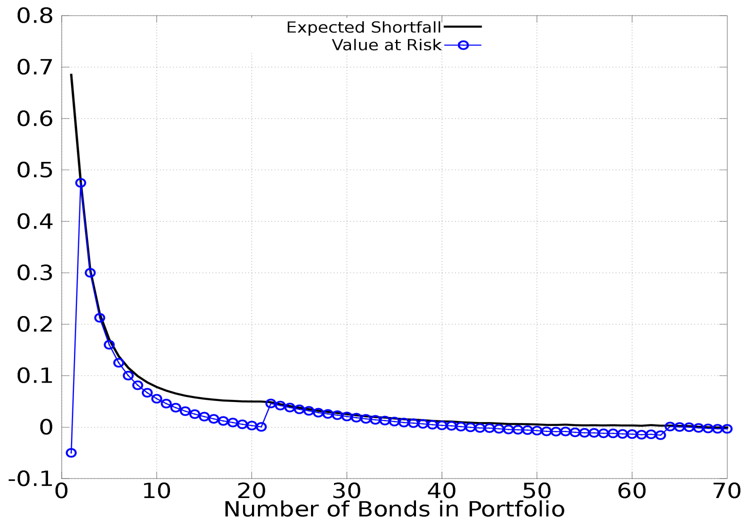

Figure 6: ES and VaR in function of number of bonds.

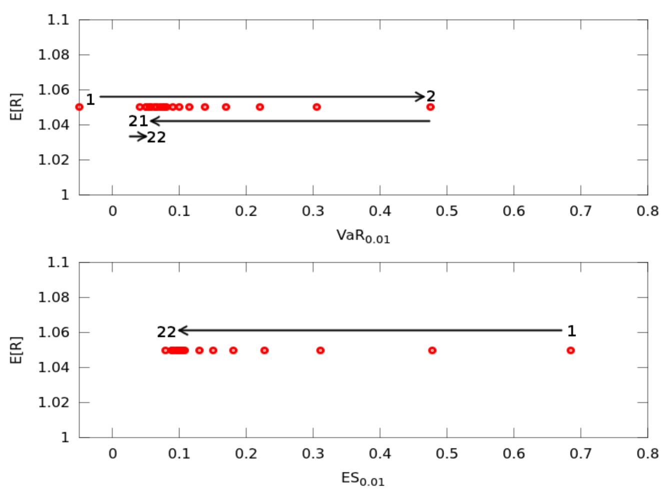

Figure 7: The result on the risk surface.

The numbers we have chosen to include in the examples are:

$$\boldsymbol{\mu} = \begin{pmatrix} 0.12 \\ 0.06 \\ 0.02 \end{pmatrix}$$

$$\mathbf{\Sigma} = \begin{pmatrix} \text{4.0000e-02} & \text{-3.1100e-04} & \text{-3.8000e-05} \\ \text{3.1000e-04} & \text{1.5000e-02} & \text{-4.7000e-05} \\ \text{-3.8000e-05} & \text{-4.7000e-05} & \text{4.2000e-03} \end{pmatrix}$$

This example is already a stylized version of reality, and a Black-Litterman approach is not really needed. If, for example, we were to choose the following naive market weights

$$\boldsymbol{w_{market}} = \begin{pmatrix} 0.33333 \\ 0.33333 \\ 0.33333 \end{pmatrix},$$

then it would only lead to minor changes in the implied return vector (in the absence of any specific market view)

$$\boldsymbol{\mu_{BlackLitterman}} = \begin{pmatrix} 0.132170 \\ 0.050877 \\ 0.01717 \end{pmatrix}$$

The precise definition of return (i.e. the log-returns or percent-returns) may be dependent on the context where these examples are used (or may be unimportant).

However, in the simulations we will assume for simplicity that the log-returns of these asset classes are normally distributed, and that the portfolio values are hence log-normally distributed.

In the case that we explore investment horizons $T > 1$, then we will make the additional assumption that the portfolio value is normally distributed.

The Hedge Fund On average, hedge funds typically display quite good information ratios, but they still have very unlikely probabilities of large losses. This is a typical example where the normal distribution fails to capture the essence of the dynamics, and where therefore the mean-variance method (MV) also fails.

Although MV in itself does not assume a normal distribution,[2] by using variance (or standard deviation) as the sole risk parameter, it fails to capture essential parts of the dynamics.

In this case, the MV approach will be misled by the small variance of hedge funds, and will ignore the significant tail risk. Using Expected Shortfall naturally captures at least part of the tail risk.

The hypothetical hedge fund (HF henceforth) used in our simulations is an overlay of two normally distributed processes.

$$f_{HF}(R) = a\frac{1}{\sqrt{2\pi{\sigma_1}^2}}\exp\left\{-\frac{(R-\mu_1)^2}{2{\sigma_1}^2}\right\} + (1-a)\frac{1}{\sqrt{2\pi{\sigma_2}^2}}\exp\left\{-\frac{(R-\mu_2)^2}{2{\sigma_2}^2}\right\}$$

[2] MV only takes the stance that return is good and volatility is bad. This is not assuming normality, but the approach fails to capture the richness of the dynamics when distributions are not normally distributed. Although insufficient to solve such real investment problems, MV remains a valid framework.

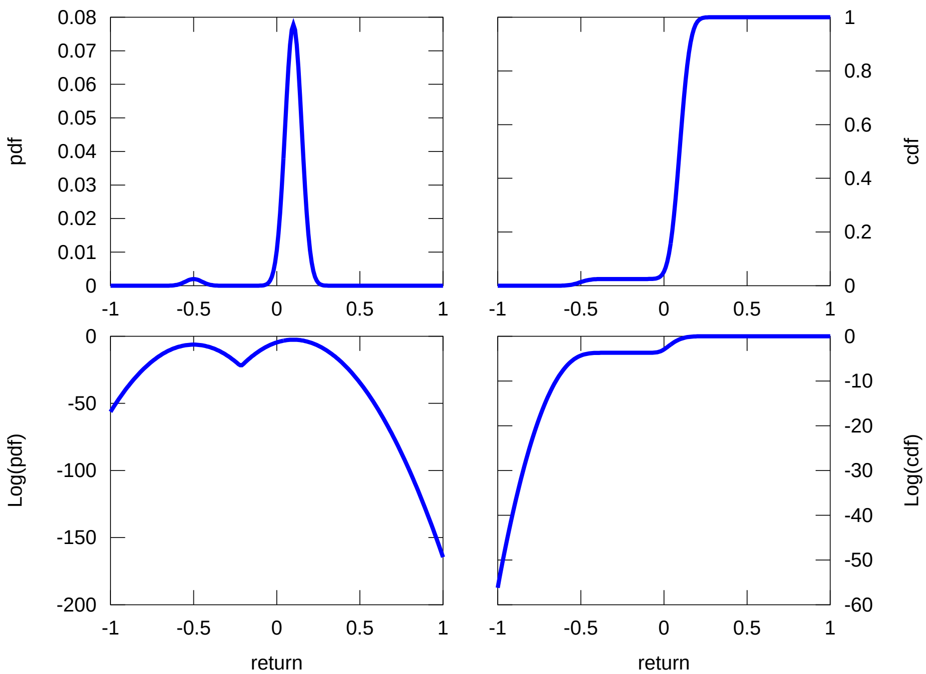

Figure 8: The distribution of the hypothetical hedge fund: on the upper row we have pdf and cdf, and on the lower row the same but with a logarithmic scale, in order to show better the effect of the small probability on very low returns.

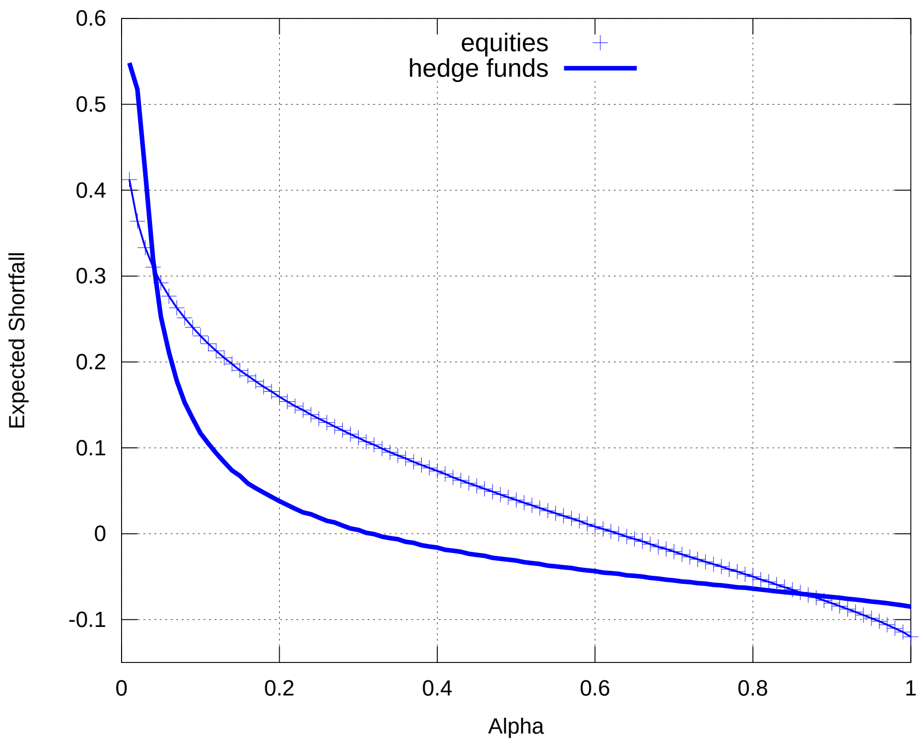

Figure 9: This graph shows the Expected Shortfall (solid line) for the hedge fund and for the equities (line and crosses). For very large levels of confidence (small α, in our case smaller than 4.2%), the hedge funds display a larger Expected Shortfall.

where we have chosen the following parameters: $\begin{cases} a & = 0.975 \\ \mu_1 & = 0.10 \\ \sigma_1 & = 0.05 \\ \mu_2 & = -0.50 \\ \sigma_2 & = 0.05 \end{cases}$

This distribution is shown in Figure 8, and results in very favourable information ratios compared with the asset classes presented in the previous section, but obviously also in an important downside risk that is not captured when only the second moment is considered!

The Relevance of the Hedge Fund. In order to study the impact of the non-normal distribution and its effect on portfolio optimization, we will consider the following simplified model. The hedge fund will be mixed with a diversified portfolio consisting of one third of each of the classical asset classes presented above.

	the α Expected Shortfall					
α	0/1	0.2/0.8	0.4/0.6	0.6/0.4	0.8/0.2	1/0
0.00309	0.58261	0.45919	0.34777	0.24858	0.16305	0.18115
0.01000	0.54829	0.42956	0.31741	0.21243	0.12934	0.15066
0.03236	0.39491	0.30578	0.22057	0.14026	0.09281	0.11601
0.10472	0.11359	0.08319	0.06287	0.05297	0.05285	0.07497
0.33889	-0.00416	-0.01221	-0.01312	-0.00642	0.00525	0.02194
μ	0.08500	0.08015	0.07629	0.07244	0.06859	0.06573
σ	0.10618	0.08649	0.07152	0.06464	0.06834	0.08119

Table 3: This table presents an overview of the expected shortfall of the hedge fund, mixed with a traditional portfolio. The upper section concerns the expected shortfall, with α in the first column, and different mixes between the hedge fund and the traditional portfolio (consisting of one third each of cash, bonds, and equities) in the rest. The numbers in the headings are respectively the allocation to the hedge fund and the allocation to the traditional portfolio. For reference, the two last lines give the volatility and the expected return for the different mixes.

The table shows the importance of working with a coherent downside risk measure, and the exceptional power of expected shortfall in particular. For high levels of certainty (small α), we find that the common perception that hedge funds are risky is indeed a correct one. There is no additional benefit in diversifying our (already diversified) portfolio further with the inclusion of hedge funds. The ES_α is a monotonously increasing function of the percentage allocated to the hedge fund for levels of α up to 1%.

For levels of α of 2.5% and above, we notice that it is possible to decrease the ES by allocating any percentage to the hedge fund. For example, assume that

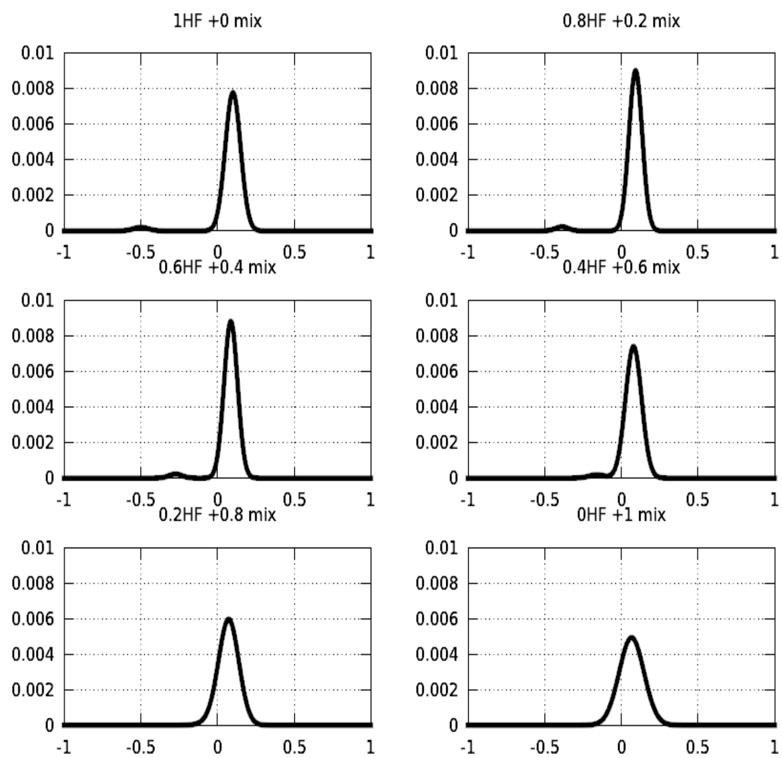

Figure 10: This figure shows the effect on the pdf, considering portfolios with different allocations to the hedge fund ("HF"), with the composition of the rest ("mix") being kept constant at one third each of cash, bonds, and equities. The leftmost graph has only the hedge fund, and the rightmost graph has no hedge-fund allocation. The same compositions are used as in Table 3.

we are interested in a 5% expected shortfall: then a 30% allocation to the hedge fund is optimal.[3]

An analysis that only takes volatility into account (and hence disregards all specificities expressed by the fact that the distribution is non-normal), would always allocate a significant part of the portfolio to hedge funds. This is because the hedge fund has an excellent trade-off between volatility and return. In the table—or from a plot of the data—we see that the four portfolios with the highest allocation to the hedge fund dominate the other portfolios in a mean-variance sense! In a mean-variance analysis, we find that all investors (for all targets, but for the one-year investment horizon considered here) should allocate at least 40% to hedge funds!

This can be taken as another example of the importance of working with coherent risk measures. Once more we find that the results obtained by using expected shortfall are coherent with our intuitions of financial risk.

A corollary of this might be that any enforcement of a maximal risk level (now done via VaR limits, or minimal diversification rules for UCITS funds, for example), could also focus on one simple risk budget expressed in terms of expected shortfall. That risk measure would automatically filter the important aspects out from the irrelevant ones, and one rule could thus replace many.

A Structured Investment We assume a simple structure that offers capital guarantee, and is based on a call option. Its pay-off structure would then be $\max\{0, R_{ref}\}$, where R_{ref} is the return of a certain reference index. Typically this will be an index in which dividends are not re-invested, and hence we will build the distribution function of this structured fund on a reference return that is lowered by 3%, in order to make up for this dividend difference. Given our parameter choice, the equities have a volatility of 20%, and the interest rates are at 2%—but these are net interest rates, and so the nominal interest rates (before inflation) might be, for example, 4%. Under such conditions—assuming a Black-Scholes market and using the Black and Scholes (1973) formulae, the price of an at-the-money (ATM) call would be roughly 10% of the nominal investment, while an interest rate of 4% would then only allow purchasing about $\frac{2}{5}$ of the nominal amount of the ATM-call option.[4] The result is that our structure will have 40% leverage on the upside of the index (and capital protection on the downside). All this means that the pay-off of the structured investment is

$$\Pi := \Lambda \left[\, R_{ref} - \mathrm{DY}\right]^{+}$$

With the leverage, Λ, at 40%, and the dividend yield, DY, at 3%.

[3]It is of course important to remember that this is an academic example, whose results should not be extrapolated to reality. Further, it may well also be useful to perform more simulations around the levels of interest with less interpolation. We choose, however, to present a concise table and focus on the interpretation.

[4]Of course, there are plenty of other possible pay-off structures. For example, one could choose to keep the 100% participation in the index return for the first 7%, and short a call with a strike price of 107%, effectively topping the maximum return at 7%. Another possibility would be to choose an out-of-the-money call with a strike price of about 88%: this option would keep the upside potential intact, but would introduce a maximal loss of 12%.

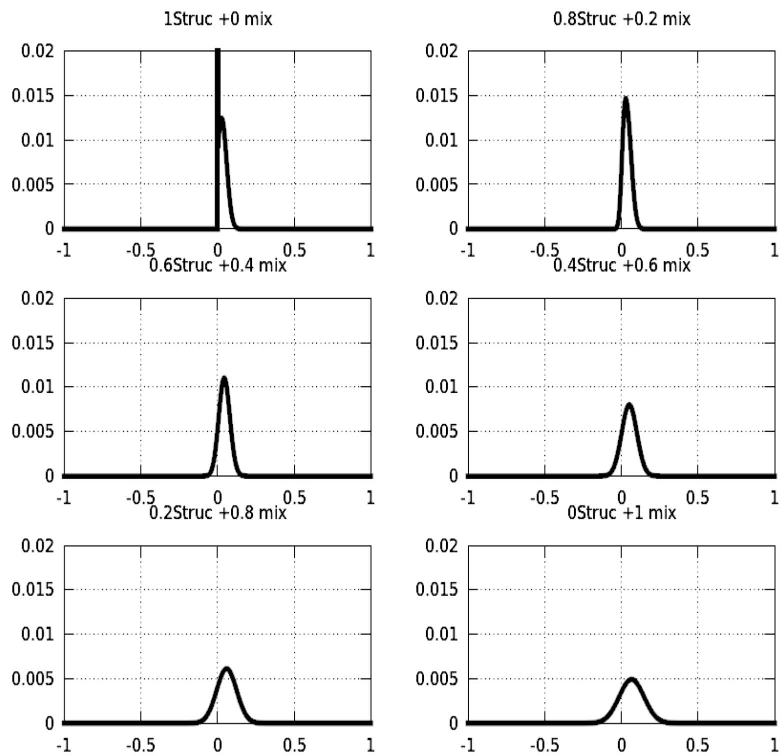

Figure 11: This figure shows the effect on the pdf of the portfolio when different allocations to structured funds (“struc”) are considered, and the composition of the rest (“mix”) is kept constant at one third each of cash, bonds, and equities. The upper-left graph shows only the structured fund, and the lower-right graph has no structured fund allocation. The same compositions are used as in Table 4.

	Expected Shortfall					
α	0/1	0.3/0.7	0.4/0.6	0.6/0.4	0.7/0.3	1/0
0.00309	0.00000	0.03160	0.06375	0.09851	0.13750	0.18115
0.01000	0.00000	0.02488	0.05061	0.07945	0.11285	0.15066
0.03236	0.00000	0.01697	0.03545	0.05771	0.08484	0.11601
0.10472	0.00000	0.00710	0.01700	0.03176	0.05160	0.07497
0.33889	-0.00011	-0.00760	-0.00813	-0.00220	0.00864	0.02194
μ	0.05357	0.05507	0.05757	0.06006	0.06256	0.06573
σ	0.05839	0.04940	0.04769	0.05395	0.06596	0.08119

Table 4: This table presents an overview of the expected shortfall of the structured fund mixed with a traditional portfolio. The upper section concerns the expected shortfall, with the α in the first column, and different mixes between the structured fund and the traditional portfolio (consisting of one third each of cash, bonds, and equities) in the rest. The numbers in the headings are respectively the allocation to structured funds, and then the allocation to the traditional portfolio.
For reference, the two last lines give the volatility and the expected return for the different mixes.

To some extent, the results of including a structured fund, based on a long call, are opposite to the analysis of the hedge fund. Where the hedge fund introduced a high return, but a non-negligible probability of having a very low return, the structured fund blocks low returns (below zero in this case). If one is interested in really low probabilities of having a return lower than a certain threshold, then structured funds are an optimal choice.

Similar to the results, there is a certain level of confidence, $(1-\alpha)100\%$, below which only the structured fund would be interesting. In table Table 4, we see that at a 90% confidence level ($\alpha = 0.1$), it is no longer optimal to have only structured funds in the portfolio. For example, a portfolio with 30% invested in the mix of cash, bonds, and equities has a lower expected shortfall.

This illustrates that if the focus is on risk control, then structured funds are a reasonable investments. However, in practice there are a few things that make the real situation less favourable with these products.

For example,

- there are the additional costs of structured funds: typically there is a management fee which is disclosed in the prospectus, plus the option writer's undisclosed profit margin,
- there is a considerable entrance fee, and these products are less liquid (monthly to weekly liquidity, compared to an equity fund that typically has daily liquidity),
- these structured products are generally not available for the investment

horizon relevant to the investor, and if they are tailor made to match the investor's horizon, then they are almost illiquid (and will be less cost effective),

- this is an analysis for an investment horizon of one year: the results are dramatically different for longer investment horizons.

Probably the most important aspect is that the results are different when longer investment horizons are considered. This mechanism, whereby the downside risk shifts when the time horizon increases, is described for example in Samuelson (1963) and De Brouwer and Van den Spiegel (2001). This actually means that if the investment horizon is only one year, then only very safe investments should be considered.

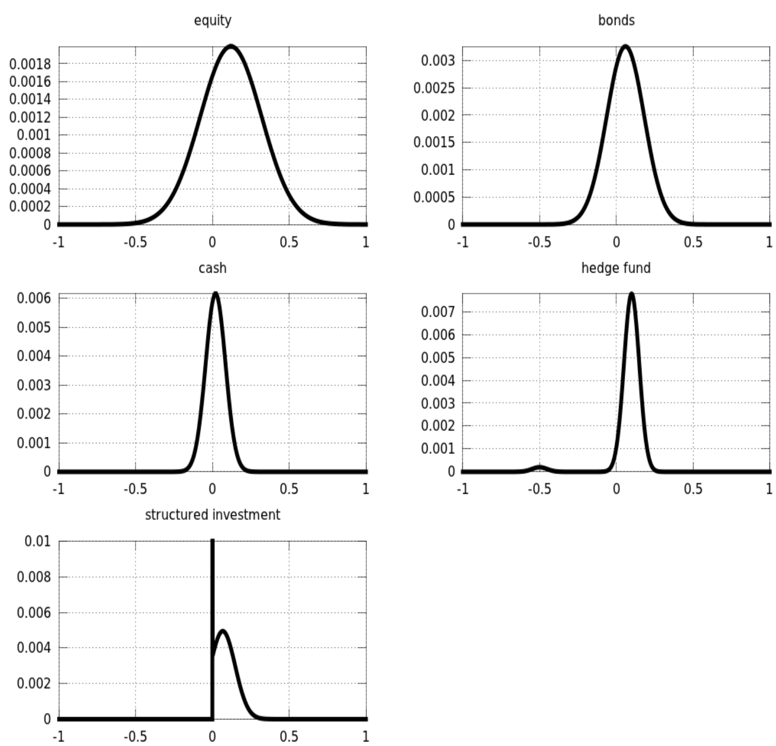

Figure 12: An overview of the probability density functions of the assets used in the examples. Note that the pdf of the structured investment is truncated.

3.4 Additional Assumptions

By adding these assets, we make the following assumption on the correlations in the covariance matrix. Note that the variances and the returns of the new assets are directly derived through their construction. It is sufficient to calculate the first and second moment from the distribution functions, in order to calculate these parameters.

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.04000 & -0.00031 & -0.00004 & 0.00035 & 0.00250 \\ 0.00031 & 0.01500 & -0.00005 & 0.00035 & 0.00010 \\ -0.00004 & -0.00005 & 0.00420 & 0.00005 & 0.00010 \\ 0.00035 & 0.00035 & 0.00005 & 0.01124 & 0.00020 \\ 0.00250 & 0.00010 & 0.00010 & 0.00020 & 0.00449 \end{pmatrix}$$

$$\boldsymbol{\mu} = \begin{pmatrix} 0.12000 \\ 0.06000 \\ 0.02000 \\ 0.08500 \\ 0.07600 \end{pmatrix}$$

where the order of the assets is in both cases: equities; bonds; cash; hedge fund; structured investment.

Case 6
Risk-Reward Optimization for Gaussian Returns

Example 11 (Three Gaussian Assets). Consider three assets (or asset classes) that are all Gaussian (or at least elliptically) distributed and consider a risk-reward optimization

Case 6
Optimal Portfolio Composition

Would VaR, ES and VAR be different here?

Question: Would VAR (σ) optimization yield different portfolio compositions

(A) Yes

(B) No

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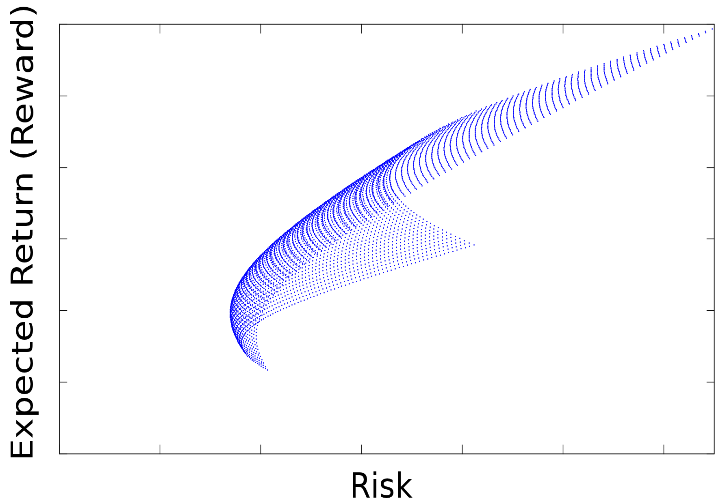

Figure 13: Portfolios in the risk/reward plane.

Case 6

Gausian Equities, Bonds and Cash—inflation adjusted

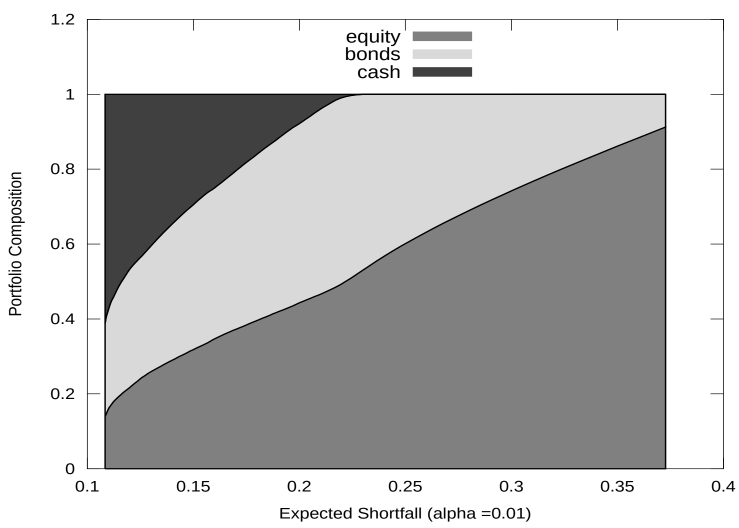

Figure 14: Recommended portfolios in function of ES.

Note that for Gaussian assets σ, VaR and ES lead to the same optimal portfolios.

3.5 Risk Reward for Non-Gaussian Assets

Case 6

Risk-Reward Optimization for Non-Gaussian Returns

Example 12 (Non-Gaussian Assets). Consider three assets (or asset classes) that are all Gaussian distributed and consider a risk-reward optimization, but add a typical hedge fund and a typical capital guaranteed structure.

Case 7: Non-Gaussian Assets

The pdfs

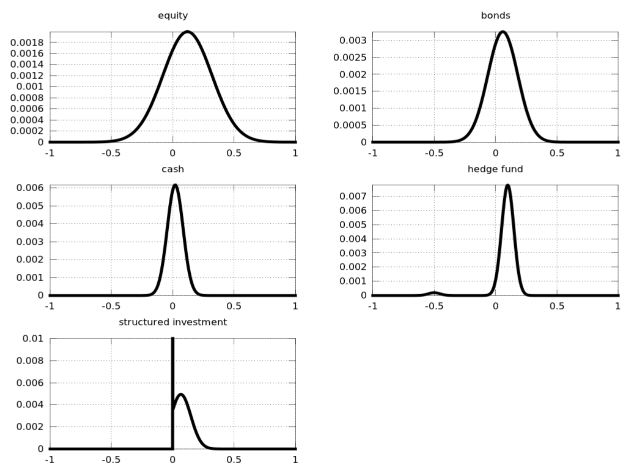

Figure 15: The pdfs in the example (the y-axis for the structured fund is truncated — this is a long call plus a deposit).

Case 7: Non-Gaussian Assets

Mean-ES and Mean-VAR Optimization

The minimum variance porftolio has a 17.5% probability to have negative returns, while the minimum-ES portfolio has a 0% probability on negative returns. For more details, see De Brouwer, 2012 – pg. 258

3.6 VaR as Risk Limit (e.g. UCITS IV)

VaR as Risk Limit in UCITS IV

For UCITS that are not managed relative to a benchmark UCITS IV defines the "Absolute VaR" limit:

$$VaR_{UCITS} \leq 20\% NAV$$

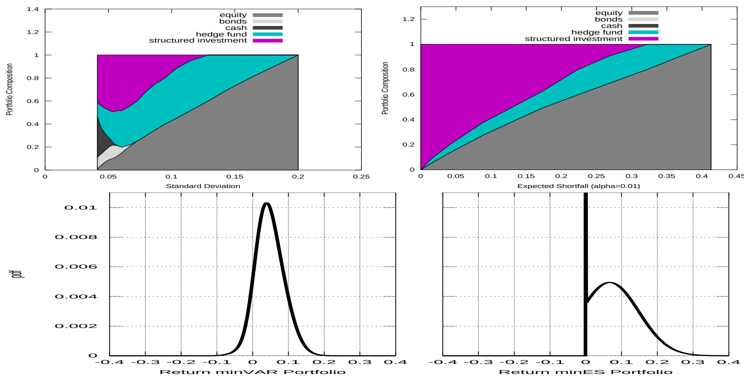

Figure 16: The min-VAR and min-ES portfolios compared.

Details are published in the General Guidelines of CESR/10-788

Case 7
VaR as Risk Limit (UCITS IV) - Risky Fund?

Question: Risky Bet Fund

Consider a structured fund that will pay in one year time 105% of the initial investment (assume that it pays the capital back plus a coupon of 5% in one year), except if company X defaults in that year, then it pays 0% (all is lost). We estimate the probability that company X defaults in one year to equal 0.7%.

(A) The VaR_{UCITS} is -5%, so this is perfectly acceptable.

(B) The VaR_{UCITS} is -5%, so this is **not** acceptable.

(C) The VaR_{UCITS} is higher than 20%, and hence it is **not** acceptable.

Case 8
VaR as Risk Limit (UCITS IV) - Better Fund?

Question: Better Diversified Fund

Consider a structured fund that will pay in one year time 105% of the initial investment, if either company X or Y defaults then it pays 52.5% of the initial investment, and if both companies X and Y default then it pays zero. We estimate the default probability of both company X and Y to equal 0.7%, and their default possibility is independently distributed.

(A) This is perfectly acceptable.

(B) This is **not** acceptable.

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Note: the same holds for the VaR limit in Basel II ICAAP. Examples: Lehman Brothers, Dexia, . . .

3.7 A Risk Reward Indicator Based on Volatility

Case 9

A Risk Reward Indicator Based on Volatility (UICTS IV)

UCITS IV defines the “Risk Reward Indicator” as follows.

risk class	volatility equal or above	volatility less than
1	0%	0.5%
2	0.5%	2.0%
3	2.0%	5.0%
4	5.0%	10.0%
5	10.0%	15.0%
6	15.0%	25.0%
7	25.0%	$+\infty$

Table 5: The “risk classes” as defined by CESR in CESR/10-673, pg. 7, in the same document the risk classes are *also* referred to as “risk and reward indicator”.

Case 9

A Risk Reward Indicator Based on Volatility (UICTS IV)

Example 13 (Risk Classification). Assume the assets from Example 12 plus one "risky bond" (this could also be a structured fund based on a digital option) that has a probability of 1% to loose 15% and a probability of 99% to gain 5%. Then consider the risk class as defined by CESR/10-673. The results are as in Table 6.

Case 9
A Risk Reward Indicator Based on Volatility (UICTS IV)

portfolio	**risk class**	σ	$ES_{0.01}$
equity	6	0.2000	0.4123
bonds	5	0.1200	0.2660
hedge fund	5	0.1062	0.5482
structured investment	4	0.0671	0.0000
risky bond	2	0.0198	0.1500
mix 1/2 equity + 1/2 bonds	5	0.1173	0.2223

Table 6: The risk classes for Example 39. CESR/ESMA's method considers the hedge fund that has roughly a 2.5% probability of loosing about 50% of its value is in the same risk class as a bond fund. A structured fund that has no risk to lose something ends up in the fourth risk class, but the risky bond that has a 1% probability of loosing 15% is considered as very safe!

What do you think?

Question: What describes best your opinion about the "Risk and Reward Indicator" in UCITS IV?

(A) Excellent choice of words and excellent idea

(B) Good idea, but . . . oh God

(C) Misleading because risk and reward cannot be reduced to one number

(D) Misleading because risk and reward cannot be reduced to one number and on top of that one has chosen an incoherent risk measure

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What would you do?

Question: What would you do in the European Parliament

(A) I would propose a "Risk and Reward Indicator" based on VaR

(B) I would propose a "Risk Indicator" based on VaR

(C) I would propose a "Risk Indicator" based on ES

(D) I would propose a "Risk and Reward Indicator" based on ES

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Case 10
Bonus Example

Question: The Evil Banker's Third Dilemma

How would a truly evil banker reduce the risk class of the "risky bond" structure?

(A) Reduce the management fee

(B) Increase the management fee and reduce the maximal payoff

(C) Something else

(D) It cannot be done

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3.8 More Cognitive Dissonance in UCITS IV

Case 11
Incoherence between the VaR-limit and the VAR-risk-class

risk limit, based on VaR$\Longleftrightarrow$risk classification, based on standard deviation

Example 14. Consider a structured fund that offers a 1% probability to loose 21% and a 99% probability to gain 5%. Such fund would not be possible, because its 1% VaR_{UCITS} would be 21% (exceeding the limit and being classified as "too risky"). Its volatility is 2.5870%, that is only risk class 3, hence considered as safer than bonds—from our example, in the middle of the spectrum, and perfectly acceptable.

3.9 Legislation

Incoherent risk measures in legislation

legislation	"risk measure"	result
UCITS	VaR and VAR	non suitable assets
Basel	VaR	crisis
Solvency	VaR	insolvency

Table 7: Law makers increasingly use non-coherent risk measures in legislation, resulting in encouraging to take large bets, ignore extreme risks and mislead investors. All building up to the next crisis ... building up to the next global disaster.

4 The Limits of Coherent Risk Measures

The Limits of Coherent Risk Measures

Liquidity

Question: Will a coherent risk measure still work in case liquidity dries up

(A) Yes

(B) No

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The Limits of Coherent Risk Measures

The Limits of Coherent Risk Measures

Question: Basel II with ES?

Would it make sense to replace VaR in the capital requirements for banks by ES?

(A) No, it would not be better

(B) Yes, it would be better, but still not perfect and could still be dangerous in case of a crisis?

(C) Yes, it could be trusted

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The Limits of Coherent Risk Measures

Example 15 (Decreasing Marginal Utility). Coherent risk measures do not seem to be congruent with decreasing marginal utility. Would it make sense to relax the homogeneity axiom when modelling preferences?

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The Limits of Coherent Risk Measures

Question: Risk and Reward Indicator?

Could a coherent risk measure ever be a “risk and reward indicator”?

(A) Yes

(B) No

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5 Conclusions

Conclusions
Coherence does matter and its importance cannot be underestimated

A. Coherence does matter.

B. An incoherent risk measure will lead to counter-intuitive and dangerous results.

C. Hence, it is worth to make a rough estimate about the left tail of the distribution rather than ignoring it.

D. Also Coherent Risk measures are a simplified reduction of the complex reality

Back-matter

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Nomenclature

α — a level of probability, $\alpha \in [0, 1]$ (to characterize the tail risk α will be "small"—for example for a continuous distribution one can say with a confidence level of $(1 - \alpha)$ that the stochastic variable in an experiment will be higher than the α-quantile)

$\delta(.)$ — the Dirac Delta function: $\delta(x-a) = \begin{cases} 0 \; if \; a \neq x \\ +\infty \; if \; x = a \end{cases}$, but so that $\int_{-\infty}^{+\infty} \delta(x-a) \, \mathrm{d}x = 1$

Λ — Leverage (expressed as a percentage)

$\phi(p)$ — the risk spectrum (aka risk aversion function)

ρ — a risk measure, $\rho : \mathbb{V} \mapsto \mathbb{R}$

$ES_\alpha(\mathcal{P})$ — Expected Shortfall = the average of the α100% worst outcomes of $\mathcal{P}$; aka CVaR, Tail-VaR, etc.

$F_X(x)$ — the cumulative distribution function of the stochastic variable X

$M_\phi(X)$ — a spectral risk measure

p a probability (similar to α)

$VAR(X)$ Variance: $VAR(X) = E[X^2] - E[X]^2 = \sigma^2$

$VaR_\alpha(\mathcal{P})$ Value at Risk

ATM at-the-money, i.e. the option strike price equals the value of the underlying asset

BCE Before Common Era

DY Dividend Yield (expressed as a percentage)

HF Hedge Fund. This refers to the hypothetical example constructed, rather than to the real asset class.

pdf probability density function

WEL Worst Expected Loss