# Financial Mathematics Project: Market Risk Analysis with VaR Models

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# 1 Project Description

Despite its lack of coherence, Value-at-Risk (VaR) remains a relevant risk metric in financial practice and regulation. This project evaluates three VaR methodologies in the post COVID-19 market crisis and encourages to think about VaR, its limitations and applications.

The purpose of this project is to use basic quantitative tools used for VaR risk projections, such as:

- Historical VaR
- Gaussian VaR
- Filtered Historical VaR

**Note**: If during the courses other methods were used, then it is acceptable to use only those.

The estimators should be applied to recent market data, i.e. daily (close price) returns for the selected four stock index returns (e.g. DAX, FTSE, WIG, S&P, NIKKEI). VaR at level 1% should be computed using different lookback/estimation periods (e.g. n=250,500). The performance of each estimator should be assessed using the statistical tests proposed below and be cross referenced to major real-life events (pandemics, wars, tarifs war, etc.). Evaluation should include backtesting results, i.e. 1-day rolling-window backtest that is aligned with regulatory backtest concept.

# 2 Data Requirements

Data can be downloaded from a publicly available sources (e.g. Yahoo Finance). We recommend downloading daily closing prices and using this to create daily relative returns. The data period should be the last full 5 years.

- Choose four indices, for example S&P 500, DAX, WIG20, FTSE 100
- 5 years of daily adjusted closing prices

**Hint**: calculate log returns: $r_t = \ln(P_t/P_{t-1})$

# 3 Methodology

## 3.1 VaR Estimation Methods

- Historical VaR: Non-parametric quantile estimation
- Gaussian VaR: $\text{VaR}_{0.01} = \mu + \sigma\Phi^{-1}(0.01)$
- Filtered Historical VaR: GARCH volatility adjustments – [1]

Table 1: Key Statistical Tests

| Test | Purpose | Reference |
|---|---|---|
| Ljung-Box | Autocorrelation | [4] |
| Jarque-Bera | Normality | [5] |
| Christoffersen | Conditional coverage | [2] |
| ADF | Stationarity | [6] |

### 3.2 Statistical Tests

### 3.3 Performance Metrics

$$\text{Violation Ratio} = \frac{\text{Observed Exceptions}}{\alpha \times T} \tag{1}$$

$$\text{Quantile Score} = \frac{1}{T}\sum_{t=1}^{T}(\alpha - \mathbb{I}_{r_t < \text{VaR}_t})(r_t - \text{VaR}_t) \tag{2}$$

## 4 Backtesting Framework

Aligned with Basel III requirements [3]:

- 1-day rolling window
- Traffic light exceptions zones
- 250-day minimum observation period

## 5 Suggested workflow

## VaR Project Workflow

- **Data Acquisition & Descriptive Analysis**
  - Obtain financial data, compute log-returns $r_t = \ln(P_t/P_{t-1})$
  - Visualize returns with time series plots; discuss structural breaks linked to real events (COVID-19, geopolitical crises)
  - Test IID property: Ljung-Box test, autocorrelation plots, volatility clustering analysis
- **VaR Estimator Development**
  - Propose 1% daily VaR models: parametric (Gaussian, GARCH), non-parametric (Historical Simulation)
  - Compare estimators: $\widehat{VaR}_\alpha = \{\mu_t - z_\alpha \sigma_t, F^{-1}(\alpha), \text{etc.}\}$
  - Discuss assumptions, window sizes (1yr vs 5yr), estimation tradeoffs (stability vs responsiveness)
- **Model Implementation & Risk Computation**
  - Calculate rolling VaR estimates $\{\widehat{VaR}_t\}_{t=1}^T$
  - Visualize VaR vs actual returns; highlight exceedances
- **Backtesting & Performance Evaluation**
  - Implement tests: Kupiec POF, Christoffersen's conditional coverage, or other tests mentioned earlier
  - Compare the performance of the models and relate it back to the major events that influence financial markets; analyze lookback period impact

- Quantify accuracy: Mean Quantile Score $S_\alpha = \frac{1}{T}\sum_{t=1}^{T}(\alpha - \mathbf{1}_{r_t < \widehat{VaR}_t})(r_t - \widehat{VaR}_t)$

- **Model Comparison & Enhancement**
  - Select best performer via loss functions & regulatory tests
  - Discuss limitations (non-stationarity handling, tail underestimation)
  - Suggest improvements: EVT integration, regime-switching models, hybrid approaches

# 6 Deliverables

- Python/R code with GARCH implementation
- Presentation slide deck
- Presentation of the slide deck (10 minutes)

# References


[1] Filtered historical simulation. https://vineetv.wordpress.com/2020/05/22/fhs/

[2] Christoffersen, P.F. (1998). Evaluating Interval Forecasts. https://www.researchgate.net/publication/2432264_Evaluating_Interval_Forecasts or https://ideas.repec.org/a/ier/iecrev/v39y1998i4p841-62.html

[3] Basel Committee (2019). Minimum Capital Requirements for Market Risk. https://www.bis.org/bcbs/publ/d457.htm

[4] Ljung, G.M. & Box, G.E.P. (1978). On a Measure of Lack of Fit in Time Series Models. https://doi.org/10.1093/biomet/65.2.297

[5] Jarque, C.M. & Bera, A.K. (1980). Efficient tests for normality, homoscedasticity and serial independence of regression residuals. Economic letters, vol 6 nbr 3, pp. 255–259, 1980, http://l.academicdirect.org/Horticulture/GAs/Refs/Jarque%26Bera_1980.pdf

[6] Dickey, D.A. & Fuller, W.A. (1979). Distribution of the Estimators for Autoregressive Time Series. https://doi.org/10.1080/01621459.1979.10482531