Investment Risk and Risk Classes

Philippe J.S. De Brouwer

1 October 1998

Abstract

This paper aims to present a risk classification that can be used for all investment funds and that is close to what the investor perceives as "risk". The method further tries to use as few arbitrary parameters as possible and the assumptions are limit to the maximal extend.

We present one graphical method that could be used in a discussion with the investor, and a risk classification that is so simple that it can be used on written communication. Ideally the risk class should be printed on each fund information sheet, so that investors can compare all investments over all different distributors.

Contents

1 Introduction

When Markowitz presented his theory on portfolio optimization in 1952, he needed a measure of risk. He proposed variance, since at his time it was possible to calculate, convenient and his colleagues did know its meaning. But now we are able to present our investors risk measures which are much more adapted, to their perception of the concept “risk”.

In this paper we will present two coherent methods. First we will discuss a rich visual method, which allows to compare investments, shows us the impact of investment horizon, and the interaction between risk and return. Then we will consider a possible classification (risk categories). Our classification is coherent with the visual presentation, and its roots lay in the utility theory.

We will also discuss all the details on how everything could be calculated and estimated, in order to present coherent results to our clients all over the world.

Our propositions are chosend because of the following characteristics:

- **valid**: the concepts used are very close to what the client perceives as “risk”,
- **useful** (the same concepts are used for the visual method and the classification, it is possible to use this method for all investments, it is even possible to use it as a base for a new generation of products)
- **clear** (interpretable) and not misleading: all concepts are carefully investigated and checked to make sure that it is not possible to misinterpret or confuse results,
- **mathematically coherent and exact**: all underlying estimations are rational and are explained (no mystical exceptions for special products), and the number of arbitrary parameters and choices is limited to a minumum.

2 Argumentation

Traditionally, one uses volatility to measure risk. Despite its historical importance this measure is not the only possible measure for risk, nor is it the best. In the following chapter, we will discuss some alternatives and then check every potential measure against some criteria.

2.1 Candidates for a Risk Measure

- Standard Deviation
- The lower partial moment family

- Downside Frequency (DF): the probability to have a negative return
- Average Downside (AD): the average amplitude of a negative return
- Semi Variance (SV): the variance of negative returns (normalised for the total number of observations)
- Value at Risk (VaR): the amount that can be lost given an investment horizon and a confidence level

2.2 Validity

What is "risk"? This is a deep question, and there is no such thing as "the" answer. Depending from your reality, the perception of risk might be different.

In psychological literature (e.g. John Payne, 1973) one finds that the most relevant factors are downside frequency and average downside. However very few people will ever perceive risk as volatility. Lots of other literature exists and almost all authors will indicate some downside notion when looking at investment risk.

In 1997 Olsen questioned a few hundred portfolio managers, and 47% defined risk as a large loss or return below target. Nobody actually associated risk with unexpected high returns.

Another study performed by Investment Corporation Institute questioned a few hundred investors. The results were found to be similar. 59% defined risk as the chance of losing money, 26% viewed risk as a trade-off of return and chance of loss, 8% defined risk as a failure to meet a target return, and 6% associated risk with swings in value.

This brief overview on validity indicates us that we should choose for downside concepts such as AD, DF, VaR or SV.

AD and DF however will give only part of the information, VaR and SV can be considered as a junction of both AD and DF.

2.3 Stable and Reliable

No candidate risk measure seems to outperform the others on this stability or reliability. Stability means that all measures will produce more or less the same results if calculated at a later moment.

Besides, this is an inevitable weak point for al kind of parameters, when dealing with financial markets.

2.4 Useful

A risk measure must be useful, it must imply some applications such as

- performance evaluation
- portfolio construction
- theoretical modelling
- the measure must apply as well to our classification as to our visual presentation

This criterion does not differentiate either, all remaining downside measures will satisfy the above conditions. However AD and DF turn out to be only interesting if used together, but doing so, it is not possible to deduct one risk measure.

2.5 Clear and Not Misleading

The discussion whether a risk measure should have a direct interpretation or not might be considered as open. But if applied, it rules out semi-variance: we all know the definition by now, but what does it really mean for a non symmetrical distribution? VaR, on the contrary has, a very clear interpretation: it is the minimal amount of money that one can lose given an investment horizon and a probability level (losses can be worse!).

Another aspect which should be discussed here is the way the classification is presented. Even if your method is mathematically exact and coherent, it would be confusing if we use numbers as the names of the classes (0, 1, 2, etc.), where in fact we are not dealing with an interval scale. Indeed the investor might interpret for example that "4 is twice as risky as 2", or "1 and 3 together are as risky as 4". People are all to fast to make such calculations, but in fact that would require even a ratio scale (an interval scale would not even be enough). This can easily be avoided by using alpha numeric characters or even words for the names of our risk classes.

2.6 Mathematically Coherent and Exact

This seems to be obvious, but it is important to construct the method in such a way that it can be used for all possible investments ranging from cash, to bonds, equities, funds, warrants, etc. If we do not keep this into our mind, we will end with a method which works fine for some products, but is completely useless for some others. Forcing other products into our scale will then result in mystical and misleading results.

The importance is to understand the limits and the merits of each method, and use it wisely with sufficient precaution.

2.7 Conclusions

Considering validity, we can only work with some "downside method". DF and AD only give some partial information, and therefore we are left with semi variance and value at risk. But if we want to communicate about something which we can understand and interpret, we can only consider the concept Value at Risk.

Later on we will also find out that VaR is the only measure, which allows us to present a coherent theory for both the visual method and the risk classification.

In the next chapters, we will present a visual presentation of risk and a risk classification based on VaR.

3 The visual method: the “Risk Barometer

In order to understand the method we first need to have a look at the evolution of the percentiles of the value of an investment. Next graph plots the average value and the 5th and the 95th percentile. This means that at each horizon we have a probability of 5% to be below the line S_m, and 5% to be above the line S_M.

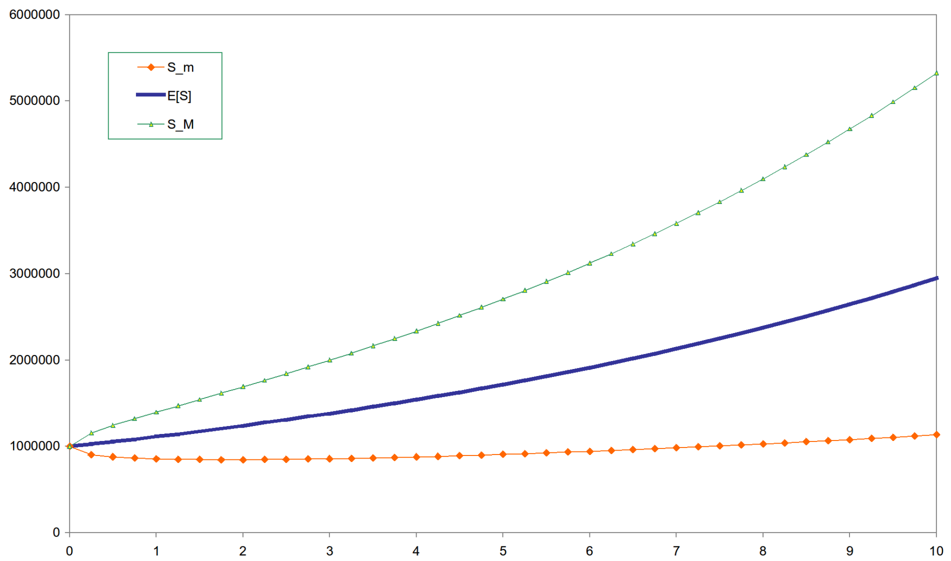

Figure 1: The evolution of the value of an investment, including $E[S_T]$ (the average value at each moment), S_m, and S_M (respectively the downside and the upside border).

In the previous graph we showed the evolution of the price of an investment. As you will know an increase of 100% over a period of two years corresponds with an annual return of 41.42%. The next graph shows these annual returns corresponding with the previous graph (total return), together with some specific time intervals.

In fact the choice of these time intervals (1, 3, 5, and 10 years) is quite arbitrary. These values, however, are commonly used in communications with the investor, and the results are, as you will see later, quite clear. These time intervals will be used for the method of visualising risk: the risk barometer.

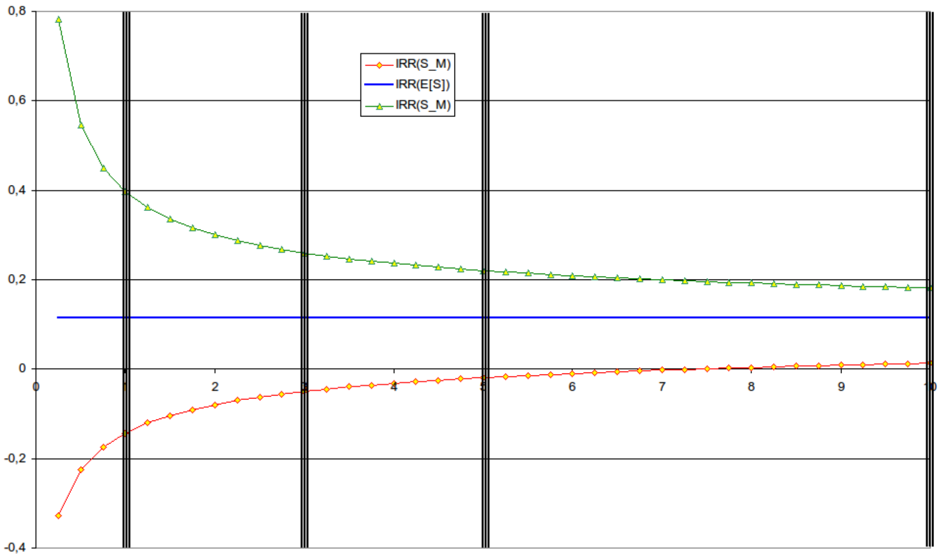

Figure 2: The evolution of the annual returns corresponding to the average value, the upside and the downside border. The 1st, 3rd, 5th and 10th year are highlighted (the vertical bars).

These graphs provide much information for an investor, but they can only visualise one investment at a time. If we would put two investments on one graph, it would result in one big jumble of lines.

The method we want to develop should be especially good in comparing different possible investments, since this is what we want to do: to help the investor to make his choice.

Therefore we look at some privileged time intervals: 1, 3, 5, and 10 years. We write down the values of the upside and the downside border for these horizons, and present these figures in one bar chart.

In the next points we enumerate some of the capacities of this method, that we will call “Risk Barometer”.

- **is a valid method**: what we present here is what the client perceives as indication of risk. In fact the downside border of our graph is the Value at Risk for the given horizon.
- **is able to compare the potential of investments and their**

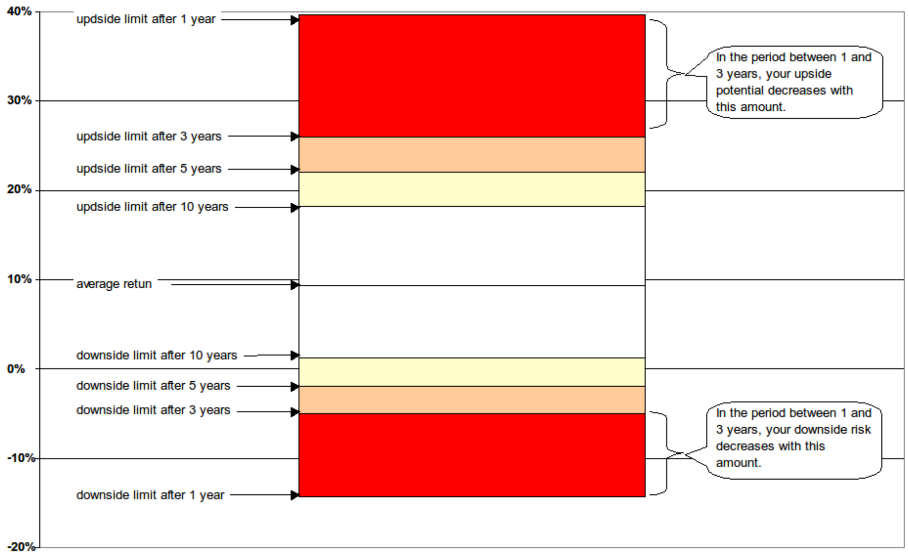

Figure 3: This figure illustrates the meaning of the “risk barometer”. Please, remember that one can expect with a probability of 90% for each horizon to find his portfolio between the two borders. It is also very important to understand that all the figures are annualised returns.

expected return: these charts are centred around the average return, by putting all investments next to each other we can compare as well downside limit, upside potential as average return.

- **is able to compare investments on volatility**: the upside and downside limit are closely related to the volatility of an investment: the higher the volatility, the wider the confidence interval.
- **visualises the interaction between return and volatility**: This is very important for the potential investor, nowadays it is common knowledge that higher expected returns are related to higher risk. But this presentation shows immediately what the result is of all particular combinations risk/return.
- **shows the impact of the investment horizon**: this is another very important aspect of investments. Downside risk will typically[1] decrease as time goes by. Even this aspect is visualised in our method: the different time horizons show very clearly that the downside risk decreases on longer investment horizons.
- **can be used for almost all investments**: indeed this unique method can be used for a savings account as well as for a government bond or a share. It is important to notice that this method can even be used for investments such as options, which have non-symmetrical statistical distributions (structured products such as capital guaranteed funds). This method even allows us to compare one share with a well diversified portfolio.
- **the method remains easy to understand** despite of its rich information content.

4 The Risk Classes

For more than one reason, it is useful to dispose of a one dimensional classification of all products. It allows for example to select investments on a transactional Internet site.

Besides of all criteria we used to select the visual method, risk classes must also satisfy some additional criteria. The concepts used and the results obtained must be 100% compatible with the Risk Barometer: we do not want any contradictions.

[1] "Typically" means here that we consider normal portfolio's and sufficient long horizons. Figure Figure 1 on page 5 shows you that the risk in terms of total return VaR will increase the first year(s): in this sense equities are more dangerous considering an investment horizon of 2 years than 3 months. But, as Figure 2 on page 6 shows, this is not true if we consider "annualised-return-VaR": there the downside risk monotonously decreases.

The concept "risk" is difficult to quantify, and we must pay attention not to create any misunderstanding. We learnt from the previous chapter that "risk" is a downside related notion, but this is not sufficient to define a classification. A risk classification must not only relate to the perception of risk, but it should also trigger for the relative importance of outcomes (possible losses and gains). In other words, we will have to answer to some extend the questions: "Is loosing 10% as half as hard as loosing 20%; and is a possible gain of 10% as valuable as a loss of 10% harms?"

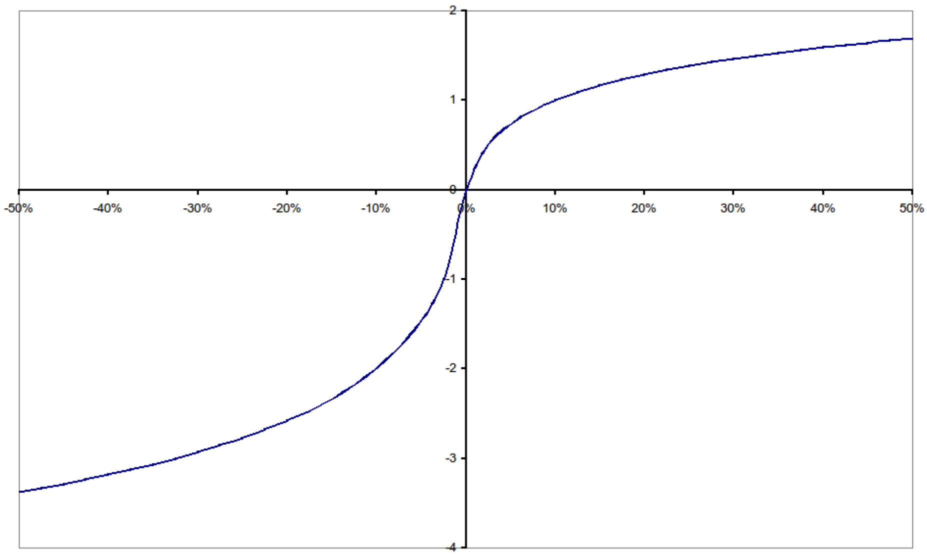

Figure 4: A typical utility function. On the horizontal axis we find the chance in % of the portfolio, and the related utility on the vertical axis.

Lots of studies have been made on that subject, but all agree that

- absolute value of losses or gains are not that important, the impact on the portfolio is determining (thus we will work in % not in $);
- a loss or a gain of the same percentage will have a different psychological impact: a loss will be considered as more important than a gain of the same amount;
- the increase of utility per unit increase of gain decreases as the gains get higher (usually referred to as the law of decreasing marginal utility);
- the decrease of utility per unit increase of loss decreases as the losses get worse, but generally the decrease of utility per unit increase of loss is more important than the increase of gain for the same gain.

It is obvious that the utility function should look more or less as presented in the previous graph. This way, the logarithmic function appears inevitably and in a natural way.[2]

This explains our choice for a risk classification.

Class	VaR (5%; 1 year)
A	negative to 0%
B	0% up to 2.5%
C	2.5% up to 10.0%
D	10.0% up to 32.5%
E	32.5% up to 100.0%

Table 1: A proposition of risk classes that are logical and use as few as possible arbitrary parameters and assumptions. Please note that negative value at risk means that the return can be expected to be positive, thus "almost no downside risk".

To create a risk classification, some choices have to be made. It is impossible to find one classification which covers all aspects of risk for every investment. A one dimensional classification is one value for one product, and risk will change in function of investment horizon.

Here we choose to base the classification on a VaR for a 1 year horizon and a confidence level of 95%. This means that this classification could also be applied to the other horizons, such as 3, 5, and 10 years horizons.

It might seem quite arbitrary, just to pick a one year horizon. However, it is a good thing that this choice is made explicit. The one year horizon is the widest, it gives us the best differentiation and it is the most distinctive on the Risk Barometer. Furthermore it is relevant and possible to make an estimation of the short term downside risk of all investments, indeed long term estimations are quite difficult (and arbitrary) for complex structures such as path dependent options.

This classification has the advantages to

- include all advantages of the visual method as far as validity is concerned, since it is based on the same concept;
- be deeply related to what an investor perceives as risk;
- closely related to the utility function of an investor;

[2]The logarithmic function appears in lots of aspects in nature, for example the scale of Richter learns us that the impact of an earthquake increases logarithmically in function of the amplitude. But especially in the perception of human senses it is merely a general rule: sound (cf. the definition of dB), light, pain (through pressure, etc.), and intensity of taste.

- be applicable for all products: even capital guaranteed funds (click-funds) can be considered, thanks to the clear (and short term downside risk related) definition used[3];
- be closely related to the visual method (excluding any conflicts);
- clear and easy to understand, and if not completely understood, it is something you can rely on;
- despite of the careful construction, we are modest and do use alphanumeric characters and not numbers to characterise the risk levels. We could argue that in this very particular case we would be allowed to use numbers, but remember that the utility function is something average, and that particular individuals will interpret our system.

A calculation of VaR is always based on four[4] parameters: the expected return, the expected volatility, and a given time horizon and confidence level. The last parameters (the time horizon and the confidence level) are chosen to be one year and 5% respectively, but the two others are just what they are: estimations. Besides, the confidence level indicates clearly that we are not dealing with sure outcomes.

5 Underlying Calculations and Estimations

As mentioned before the value at risk depends (in our approach) on two variables and two parameters. The two parameters (time horizon and confidence level) are to be chosen, but the two variables (the expected return and the expected volatility) are to be determined somehow.

5.1 Expected Returns

Of course, we considered the use of particular predictions or historical averages as expected returns. But we are also convinced that this information on risk should be independent from our own (temporarily) predictions (which we have to make as an investment manager). And historical performance is seldom a good predictor for the future behaviour in financial markets.

To make rational, easy-to-verify, and consistent predictions we will use the "method-with-the-risk-premium". The general idea is that you take relevant interest rates and yields-to-maturity to predict the return of cash and bonds, and that you add a risk premium for equities and some classes of bonds in order to obtain the expected return.

[3] A fund which guarantees the initial capital, can for example have a class B (because the risk metric is calculated on 1 year horizon; and on that short horizon fluctuations might occur).

[4] Assuming that we know the statistical distribution law to be Gaussian.

The risk depends on the currency in which the investment is expressed and the “home currency” for the particular investor. For European investors, this home currency will be the euro. In the next table you will find the ways that are used to estimate the expected return of the different asset classes. If we are dealing with an investment which does not have any benchmark, usually the product itself can be taken into account, or another proxy.

Asset Class	**E[R] in euro**	**E[R] in local currency**
Cash	interest rate in euro	interest rate in local currency
Bonds	yield of the benchmark + x%	yield of the benchmark in local currency + x%
Equities	yield 10 years in euro + y%	yield 10 years in local currency + y%

Table 2: Assumptions for the asset classes using as few as possible arbirtrary parameters and avoiding recent history bias.

the following risk premiums could be used

- x% = 0.0% for government bonds and 0.5% for corporate bonds
- y% = 3.0% for all equities, except 5.0% for emerging markets

If we are dealing with a mixed fund, the weighted expected return will be calculated using the benchmark of the fund and the weights of each asset class.

This would indeed be a simple and straightforward approach, it has the virtue of being simple, but it lacks a good theoretical backing and it introduces some additional parameters. It is theoretically much more appealing to use a Black-Litterman approach to estimate the expected returns. In that model, the expected returns are modelled via the market weights and the observed covariances. The Black Litterman approach is better because it also allows a finer grain in the expected returns (something that is needed when many investment funds have to be compared). Furthermore it uses implicitely the the aggregate knowledge present in the financial markets.

These predictions for the one-year return will be denoted $E[R]$. This is an estimation for what the return will be on one year. To be used in further calculation, we need to change from this discrete definition to a continuous return (μ). This is performed as follows:

$$\mu = log(E[R] + 1)$$

5.2 Expected Volatilities

Another parameter to be estimated is the volatility of each investment. Volatility estimations will initially be based on 5 year, monthly historical data.

Asset Class	E[vol] in euro based on	E[vol] in local (other) currency based on
Cash	cash in euro	cash in local currency
Bonds	historical data in DEM	historical data in local currency
Equities	historical data in euro	historical data in local currency

Table 3: Assumptions for the asset classes using as few as possible arbitrary parameters and avoiding recent history bias.

This data will show us a volatility for monthly data (σ_{month}), to change this into annual volatility, we multiply this by the square root of twelve.

$$\sigma = \sqrt{12}\sigma_{month}$$

Since the expected returns are estimations for the future, independent of the past, it is not that coherent to use these results without any interpretation. We will use this historical volatilities as a first raw estimate of the future vol. In our calculation however we will use the implied volatility in the sense of Black-Litterman (without any specific market assumptions)

5.3 Putting the pieces back together

Now that we have determined all necessary parameters, we are ready to calculate the VaR, if we assume that the logarithmic returns follow a Gaussian[5] distribution law.

In symbols:

$$\text{define: } R = \log\left(\frac{S_t}{S_{t-1}}\right)$$

$$\text{and assume: } \quad R \propto N\left(\mu - \frac{\sigma^2}{2}, \sigma\right)$$

(i.e. the return R is normally distributed)

If we denote Φ as the inverse, standardised normal cumulative distribution, then we get the following formulae for our downside border (S_m), expected value ($E[S]$) and upside border (S_M), relative to a confidence level of x% in terms of value of the investment

[5] The "Gaussian law" is also known as the "Normal Distribution".

$$
\begin{aligned}
S_m &= S_0 \exp\left\{\left(\mu - \frac{S_t}{S_{t-1}}\right) T + \Phi\left(\frac{1-x}{2}\right)\sqrt{T}\sigma\right\} \\
E[S] &= S_0 \exp\{\mu T\} \\
S_M &= S_0 \exp\left\{\left(\mu - \frac{S_t}{S_{t-1}}\right) T + \Phi\left(\frac{1+x}{2}\right)\sqrt{T}\sigma\right\}
\end{aligned}
$$

T is the time in number of years, S_0 is the initial value of the investment. But more particularly, we are interested in the corresponding annualised returns, to plot on our graph for the visual method, so we get:

$$
\begin{aligned}
r_m &= \exp\left\{\left(\mu - \frac{S_t}{S_{t-1}}\right) T + \Phi\left(\frac{1-x}{2}\right)\frac{\sigma}{\sqrt{T}}\right\} \\
r &= \exp\{\mu\} - 1 = E[R] \\
r_M &= \exp\left\{\left(\mu - \frac{S_t}{S_{t-1}}\right) T + \Phi\left(\frac{1+x}{2}\right)\frac{\sigma}{\sqrt{T}}\right\}
\end{aligned}
$$

We choose x% to equal 90%. One particular application of these formulae is the VaR with a one year horizon and a 95% confidence level, to be used in our risk classification[6]:

$$
VaR(5\%, 1\text{yr}) = \exp\left\{\mu - \frac{\sigma^2}{2} + \Phi(5)\sigma\right\} - 1
$$

6 Conclusions

This presentation of risk offers us a complete and coherent range of instruments to provide some real added value to investors. In stead of asking "what is your risk profile?", we are now able to confront the investor with the consequences of his or her choices, and provide feed-back in a way that the investor really learns more about investing and understands better why he or she should prefer one or another investment.

Both the visual presentations and the risk classes are easy to understand. They are not only based on past performance, because that can be very misleading (and as we know "past performance is no indication for the future"). They are rather based on some rational assumptions such as risk premium and observable variables such as interest rates or they can be derived via a Black-Litterman approach. But it is extremely important to understand that despite all the efforts and the mathematical rigour that they are just

[6]It might be useful to remark that the x% used above is the confidence level for a two-tail interval, and now we only need a one-tail interval. This means that the division by 2 is not needed any more.

what they are: estimations. And be aware that a value at risk with a confidence level of 5% is not a guarantee! There is a 5% chance that things turn out to be worse . . . and VaR does not learn us how bad it can get!

It is our conviction that the ideas described in this text, present a unique way to have a coherent visualisation and classification of products based on their risk, that really point the client to what he or she perceives as risk.