Risk Management in Project Finance

by

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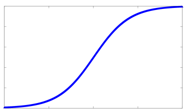

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Contents

Nomenclature

α a level of probability, $\alpha \in [0,1]$ (to characterize the tail risk α will be "small"—for example for a continuous distribution one can say with a confidence level of $(1-\alpha)$ that the stochastic variable in an experiment will be higher than the α-quantile), page 186

$\boldsymbol{\imath}$ a column vector where each element equals one, page 142

$\boldsymbol{\Sigma}$ the varcov matrix, page 142

$\boldsymbol{w}$ the vector of weights of rank M, so that $\sum_{i=1}^{M} w_i = 1$, page 140

$\boldsymbol{x}'$ the transpose of the vector or matrix $\boldsymbol{x}$, page 140

$\delta(.)$ the Dirac Delta function: $\delta(x-a) = \begin{cases} 0 \; if \; a \neq x \\ +\infty \; if \; x = a \end{cases}$, but so that $\int_{-\infty}^{+\infty} \delta(x-a)\,\mathrm{d}x = 1$, page 186

$\mathbb{R}$ the set or real numbers, page 180

$\mathbb{V}$ the set of real-valued stochastic variables, page 180

$\mathbb{N}$ The set of natural numbers = {0, 1, 2, . . . , n, . . . }, page 157

$\mathbb{R}$ the set of real numbers, page 196

$\mathcal{L}$ a loss variable expressed in monetary terms, page 148

$\mathcal{P}$ a profit variable expressed in monetary terms, page 148

μ the average or expected value of a stochastic variable X, page 182

μ the average or expected value of a stochastic variable x, page 140

μ_p the expected return of portfolio p (consisting of M loans), page 142

$\phi(p)$ the risk spectrum (aka risk aversion function), page 186

ρ a risk measure, page 181

ρ the correlation coefficient), page 142

ρ_{ij} the correlation between asset i and asset j, page 142

σ_i the standard deviation of the return of asset i, page 142

σ_p the (expected) variance of portfolio p, page 142

σ_{ij} the covariance between asset i and asset j, page 142

c a real number: $c \in \mathbb{R}$, page 143

$ess.inf acerbi2002expected$ essential infimum, page 161

F(x) the cumulative distribution function, page 147

$F^{-1}(\alpha)$ the inverse of the cumulative distribution function, by definition left continous, page 147

$f_R(t)$ the density function of a continuous distribution of a stochastic variable R, page 140

$f_X(t)$ the probability density function of a continuous distribution of a stochastic variable X, page 182

$F_X(x)$ the cumulative distribution function of the stochastic variable X, page 186

$f_{est}(x)$ the estimator for the probability density function, $f(x)$, page 168

$f_{est}(x; h)$ the estimator for the probability density function for a kernel density estimation with bandwidth h, page 168

h the bandwidth or smoothing parameter in a kernel density estimation, page 168

i counter, page 140

i	counter, page 182
inf	infimum, page 161
$int(.)$	used as a function, $int(.)$ extracts the integer part of its argument, page 157
K_h	the kernel (of a kernel density estimation) with bandwidth h, page 168
$M_\phi(X)$	a spectral risk measure, page 186
p	a probability (similar to α), page 186
$q_\alpha(X)$	the α-quantile of the stochastic variable X, page 182
$Q_X(p)$	the quantile function of the stochastic variable X, page 184
$Q_X(\alpha)$	the quantile function of the stochastic variable X, page 147
$q_{(\alpha)}$	the α-quantile, page 147
R	return, page 139
R_p	the return of portfolio p (consisting of M loans), page 142
S	semi-variance, page 144
sup	supremum, page 161
X	a real valued stochastic variable, representing a profit variable, page 182
x^-	$\begin{cases} x & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}$, page 144
ABS	Asset Backed Securities, page 124
AS	Average Shortfall, page 157
AVaR	Average VaR, page 157
CDS	Credit Default Swap, page 135
CF	Cash Flow, page 135
CF	Cash Flow, page 164

CFR	Cash Flow Risk, page 129
EAD	Exposure At Default, page 137
ECA	Export Credit Agency, page 122
EL	Expected Loss, page 137
ELTIF	European Long-Term Investment Fund, page 175
ES	Expected Shortfall, page 184
ETL	Expected Tail Risk, page 157
IRR	Internal Rate of Return, page 164
IRS	Interest Rate Swap, page 128
KDE	Kernel Density Estimation, page 167
LBO	Leveraged Buy-Out, page 124
LEP	Large Electron Proton Colider, page 126
LGD	Loss Given Default, page 137
LRL	Limited Recourse Lending, page 114
MCDA	Multi Criteria Decision Analysis, page 119
MISE	mean integrated squared error, page 169
NPV	Net Present Value, page 164
p2p	"peer to peer", page 194
P&L	Profit and Loss, page 165
PD	Probability of Default, page 137
pdf	probability density function, page 138
SLA	Service Level Agreement, page 119
SPV	Special Purpose Vehicle, page 114
TCE	Tail Conditional Expectation, page 157

VAR	variance, page 182
VaR	Value at Risk, page 148
VaR	Value at Risk, page 182
VB	Visual Basic, page 165
WPM	Weighted Product Method, page 119

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Part I

Risk Management

Chapter 1

Introduction

Large infrastructure works such as building highways, airports, power stations, harbours are most important tools to improve directly comfort and well-being or indirectly by fueling economic development. The World Bank, for example, estimates that a ten percent increase in infrastructure projects leads to a one percent increase in GDP —(Calderon, et al. 2011); hereby confirming (Evans & Karras 1994)'s findings.

In Asia alone the global pipeline is estimated at $ 9 trillion (Beckers, et al. 2013), this is more than half the GDP of the USA. The United Kingdom currently plans to engage in roughly 500 projects that would total £ 200 billion in the next 5 year and will cost £ 500 billion by 2020 – see (gov.uk 2015, Coopers 2015).

Project Finance or Limited Recourse Finance[1] is typically defined as:

> **Definition 1.0.1 (Limited Recourse Finance).** A form of financing – typically a large and capital intensive infrastructure– project where the lender gains confidence that the borrower is able to service the loan not by its creditworthiness but via a claim on future cash flows.

[1] We will use the terms "Project Finance" or "Limited Recourse Finance" interchangeably.

We will use the words Limited Recourse Financing, Limited Recourse Lending and Project Finance as synonyms and use the shorthand notation LRL.

The reason why the lender can not grant the loan based on the balance sheet of the borrower is that the borrower is typically as Special Purpose Vehicle (henceforth SPV), that is only created for the purpose of the project and hence has no credit history and little asset other than the project.

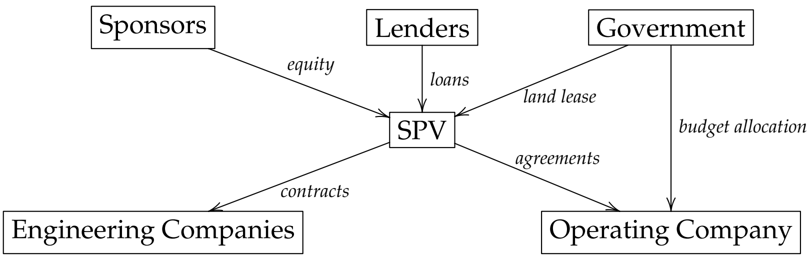

Figure 1.1: *A simplified schedule of a typical project finance setup. In reality there are more flows and agreements. For example the sponsors will need a Shareholders Agreement, the government might give credit support to the SPV and/or the Operating Company. Other variations are possible, for example the Operating Company might be a government institution, if the government is not the off-taker then no budget allocation is needed, etc. Each project is unique.*

A few definitions will make the concepts introduced in Figure 1.1 clear:

- **Government**: the government or other authority involved is the one that would like the benefits of the project to be available but lacks the financial strength and technological know-how to run the project itself. Typically the government(s) will initiate the projects and even run a tender between competing groups of companies. The Government selects the winning group by allocating to them the permission to use the land.
- **Sponsor(s)**: one or more companies that have an equity stake in the SPV. This company is hence owner of the SPV and due to their technical knowledge should be able to contribute to the management of the SPV. Typically a Sponsor is a company that has experience in executing similar projects and/or has an interest in the project being executed.
- **Engineering Company**: the Engineering company is the company that

will execute the building of the project. It might do so directly and/or subcontract.

- **Lenders**: the Lenders are the financiers of the project, they are the party that will "invest" the largest amount by granting loans, that typically are up to ten times larger than the equity investment of the Sponsors. In order to diversify these huge risks, lenders will organize themselves in syndicates and may include government agencies.
- **Operating Company**: is a general word used in Figure 1.1 on the preceding page that can be Tolling Company, the Operating Company and even the Off-Taker would fit in the same place.

The benefits are important, the amounts involved are huge and so is the complexity in all possible aspects.

Project (Country)	**planned cost (€ billion))**	**actual cost (€ billion)**	**Reason**
Betuweroute (Netherlands)	€ 2.3	€ 4.7	1.5 year delay, finally in use in 2007, partly not finalized (planned 2020)
Eurotunnel (France / UK)	€ 7.5	€ 15.0	6 month delay, 18 months of unreliable service after opening, market share gain overestimated by 200%
Rail Frankfurt-Köln (Germany)	€ 4.5	€ 6.0	unforeseen capped government spending, legal issues, 1 year delay
Kuala Lumpur Airport (Malaysia)	€ 2.0	€ 3.5	losing market share to Signapore (runs at 60% of capacity), issues with connection to downtown area, complaints about health and safety

Table 1.1: *Examples of infrastructure projects that failed to deliver in time. (source Reuters, McKinsey, Wikipedia, annual reports)*

Table 1.1 lists a few examples of infrastructure projects that did cost more more than planned. These examples are by no means exceptions. Most issues seem to be with delays related to engineering and production, inadequate forecasting of markets share, funding issues, competitive landscape, etc. Modern infrastructure projects are large and complex, but it seems to us that most of the loss of value for the society can be avoided by adequate risk management.

Larger projects typically overrun budgets or fail on a set of issues that can be summarized as:

- engineering problems delay the delivery of the project;
- the benefits of the project are overstated or do not fit in a larger strategy (for example a railroad should fit in a national mobility plan);
- financial resources do not become available as planned;
- design issues.

The good news is that therefore –it seems to us– a large part of these risks can be mitigated by a a modern, comprehensive and end-to-end risk management process that is embedded in a true risk culture. The rest of this section will try to make that point by presenting how the risk management should work and encompass all stages of the project.

Chapter 2

Integrated Risk Management and the Risk Cycle

Limited Recourse Financing or Project Finance exists mainly because (a) large infrastructure projects are necessary and useful for economic development and (b) that in order to execute these projects risks have to be shared or transferred to parties that can bear the risk better. This aspect together with the sheer size of the projects –both in time and money-terms– will make clear that risk management is one of the most important aspects of a successful project.

Since the project takes a lot of time and not in all phases the risks are the same it is important to maintain a continous cycle of risk identification, risk mitigation and risk management. Figure 2.1 on the next page illustrates this cycle of risk management. Every party involved will have to do its own risk management and use his own point of view while going throught this cycle.

The fact that involved parties can push risk from one company to another means that the risks are different for each party involved and that what is "a mitigation" for one party can be "a new risk" for another party. For example the SPV is faced with the risk of late ending of the construction phase (this will not only delay the income but it might be possible that in the meanwhile the SPV is already responsible for paying back some loans). So the SPV will mitigate that risk by adding a late delivery clause to the contracts of the engineering companies. So, now the SPV is covered while the engineering companies took that risk on board. This risk allocation seems reasonable because it is the engineering company that will execute the work and should

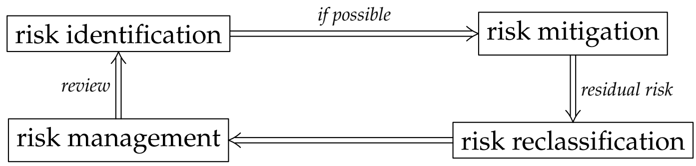

Figure 2.1: *Risk Management is an ongoing concern of utmost importance. Phase after phase, cycle after cycle and for each party one will have to go through this cycle in order to mitigate and minimize risks. Bad risk mangement is bound to lead to disaster.*

be best placed to assess the time needed to do the work and it is the only party that can act by for example adding more workers in order to assure a timely completion of the job.

Please note that in this example the late delivery risk is not completely covered for the SPV, in fact it became a risk related to the creditworthiness of the engineering company.

So, each party will have a different view on what is risk and how to manage it. What each company involved should do is create insight in the relevant risks, mitigate if possible and continuously follow up the main risks and make sure that they remain minimized. For example it is not sufficient to identify the risk that the construction phase may end late, it is only by following up every day and speeding up the process that the damage can be minimized.

Therefore each party should on an ongoing base cycle through the following aspects:

1. **Identify Risks**. Of course the main risks can be indicated by following the checklists provided in Chapter 3 on page 121, but it is necessary to review on a regular base. Some risks might have been dormant and might not have been identified at all or emerge as the fallout from the emanation of another unexpected risk. This new risk should be added to the list, mitigations have to be put in place and the risk has to be managed to keep the fallout under control.

2. **Assess the Impact and Probability**. It is not sufficient to look only at probability or only at impact when assessing a risk, it is absolutely necessary to monitor both dimensions. Also probability and impact will vary during the lifetime of a project (for example when a project is built, the risk of delayed construction falls to zero). As it is often impossible to quantify accurately these impact and probability parameters, one

typically resorts to an ordinal scale[1], such as Very Low, Low, Medium, High and Very High or simply a number between 1 and 5.[2]

3. **Mitigate Risks**. Where possible, risks will be mitigated. For example if there is a risk that the construction phase of the project will be delayed, then the SPV can put a "delayed delivery clause" in the contract and will make sure that there is a liquidate damages clause. The engineering company on its turn mitigates this risk by assuring that the deadlines are realistic, that –where subcontractors are used– their contracts reflect this clause, and –for own workers– it will make sure that that workforce is managed well and that a good risk management is in place.

4. **Calculate residual impact and probability and re-classify**. Obviously, risk will change as it is managed well. Assume that we identified the breakdown of a concrete pump as a critical risk and now we have SLA (Service Level Agreement) in place with a provider that assures us to have a new pump on our site within 48 hours. In that case the risk related to the concrete pump must be re-assessed and certainly will not be critical any more. This example show us that the risk manager who does his job well will permanently shift focus: now that the concrete pump is under control he/she will go after the next important risk and try to mitigate it too.

5. **Prioritize Risks**. Based on the above two dimensions one is able to make a classification of all risks in a one-dimensional way based on a MCDA (Multi Criteria Decision Analysis), typically something related to WPM (Weighted Product Method) is most appropriate. This prioritization can be presented as a "risk matrix" (see Chapter 4 on page 133).

Now that all the aspects are in place, it is essential to make sure that risk management becomes a cornerstone of the engagement, an ongoing concern and the focus of all. This is done by having a risk manager in place that is –preferably– at the level of the executive committee of the relevant organization and that a true risk culture is in place. It is everyones responsibility to spot risks, help to mitigate them and prevent them from realizing. When risks do have to be taken, it is worth to consider if the payoff of a given risk fits in our risk appetite (in other words if it the return is worth the risk).

[1]See Appendix B on page 197.

[2]In most cases a scale with 5 levels will work pretty well: it is not too rough (such as High, Medium and Low) and still is a lot simpler than a "real number between 0 and 1" approach. However it cannot be stressed enough that such scale is only an ordinal scale and not an interval scale and this has important impact: see Appendix B on page 197 for further information.

In the next chapters we will clarify these important steps in risk management.

Chapter 3

Risk Identification

As mentioned, Project Finance can be seen as a way to reallocate risks to the party best fit to take a certain risk on board. Each project is a delicate exercise in refining and re-organizing risk allocation up to such level that the project is both realistic and profitable for each participant. It will be clear to the reader that since Limited Resource Lending exactly exists because involved risks are too high that risk management is an essential part for each player in the setup and execution of any project.

Each participant will have his own reasons to participate and his own point of view on the risk. To understand better the deepness of this idea, we explore this from the different participant's point viewpoint.

- **the Sponsor(s)**: even large construction companies do not have the resources to finance biggest projects and if they would bring up the finance via lending for example, one failure would bring them down. Banks are by their nature better able to diversify risks (that is actually the reason why banks grew so big since the rise of the Banque de Rothschild in the era of the Napoleonic wars). Also one will notice that the Sponsors specialize in execution of similar projects and do not specialize in large financial risk taking. That is another business model.
- **the Government**: large infrastructure projects are important for the economy, however they typically span multiple political cycles. Therefore it is less appealing for politicians to do the hard work and get a project started only to see financial resources consumed and basically prepare the field for the opponent to harvest the economic advantage of the

project and the political public relations that go with the opening of the project. Therefore it is more appealing to become the "initiator" and use the financial resources for spendings that will yield shorter term payoff. Gornments can hence use LRL as an alternative to raising money on the bond market (examples of this form of funding are for example the "war bonds" in many countries — early examples are the city states in Italy in the Middle Ages). The reason to do so typically boils down to one of the twho following problems: or the country is too small and weak to take on the risk or the country is large and strong but has too man projects on the agenda. A good solution is then to use the leverage in the banking system.[1]

- **the Lenders**: the largest financial risk ends up with the lender. Of course that lender can try to mitigate risks as good as possible –as we will discuss later– but even that will typically be insufficient and therefore will seek to diversify and prefer to lend £ 50 to two projects in stead of £ 100 to one and take in each project another lender as partner and hence forming as syndicate. However –also after the Global Meltdown of 2008– this form of of collaboration became more difficult and more and more there is the tendency to resort to consortium[2] collaboration between banks. But the pressure on investment banks is so intense that they cannot longer play their natural role in our economy to the extend that our economy needs. So, more and more one resorts in putting the taxpayer's money directly at risk. This is done by copying the "off-balancing mechanism" –that leveraged banks to an unreasonable extend and was directly responsible for the Global Meltdown– to governments. The credit exposure is not kept on the balance of the government, but rather hidden in off-balance SPV's called "Export Credit Agencies" (henceforth ECAs).[3]

So, all parties involved in the project will have their own specific point of view on the project and along with that their specific risks and needs in risk

[1] Although, as we already noted the recent crackdown on banking and investment banking in particular (see for example the Basel III requirements) puts automatically and directly the risk and burden with the governments in stead of having banks as a first safety layer.

[2] The concepts of "loan syndication" and "consortium" are very close and both sport one borrower and multiple lenders that share the risk. Typically the difference is understood as such that a syndication is geared towards international collaboration (involving multiple currencies) and has one clear "managing bank" that assures a technical lead –not necessarily the largest lender.

[3] This could be seen as how a mechanism that is installed to solve one crisis typically seeds the germs for the next crisis. For example, the off-balancing mechanism (that produced the Global Meltdown) was introduced by the Clinton administration to remedy the banking problems of the eighties.

management. As the Lender is the party that risk the most money and has least control over what is happening on the field, it is an interesting point of view for risk management. Unless otherwise stated we will use in the next sections the point of view of the Lender.

3.1 Specific Types of Risk

We defined Limited Recourse Lending (LRL) in Definition 1.0.1 on page 113. From this definition follows automatically that some very specific types of risk must exist.

This implies immediately that Project Finance will have some very specific forms of risk that are typically not or rarely present in other forms of lending. We consider the following aspects as quite unique for Project Finance:

- **no credit-worthy borrower but reliance on future cash flows**: this means that the business risk is not borne by the sponsor, but rather by the lender
- **size**: projects are constructed as a LRL project, because nor the sponsor(s) nor the government are able or willing to bear the risk alone and hence the risk is transferred to investment banks[4]
- **typically for large infrastructure projects**: although sometimes universities and even prisons are build in the form of project finance, typically only durable infrastructure projects are deemed important enough.

3.2 Generic Risk Factors

Besides having its particular risk factors, that seldom appear in other projects and ventures, Project Finance also shares a large base of risk factors that are common with other types of financing. Of course, due to the size of the project the risks involved are much larger in a LRL project and some of the risks will be specific to different LRL constructions.

For example:

[4] Since the Global Meltdown (2008) banks are under much pressure to de-leverage, keep more liquidity and hence are less able to finance large and risky projects. This is of course what the regulator wants: banks had to become safer, but of course by doing so the risk is now more and more borne by Export Credit Agencies . . . so, directly by the taxpayer.

Financing Form	Similarities	Differences
corporate lending	• the structure of the loan is a "term loan" • the risk of the lender is related to the ability of the borrower to generate cash flows	• the comfort that the loan will be repaid is based on future cash flows of the project and not by the creditworthiness of the borrower • the asset that could serve as collateral does not exist yet in LRL
ABS (Asset Backed Securities)	• the borrower is an SPV • the originator ("Sponsor" in LRL) obtains "off-balance sheet financing	• the SPV issues bonds in stead of lending from bank(s) • in ABS one has a large pool of assets (eg. loans); in LRL there is one large asset
Venture Capital, Business Angels	• the borrower has (almost) no creditworthiness • the lender relies mainly on future cash flows	• in LRL the SPV is highly leveraged • the Venture Capitalist provides typically an equity stake and not a loan
LBO (Leveraged Buy-Out)	• the borrower has limited creditworthiness • both are highly leveraged transactions	• the lender has typically recourse on the borrowers, • in LBO the lender can hope that the borrower gains creditworthiness

Table 3.1: *Similarities and differences with other forms of financing.*

- **Feasibility Risk or Technology Risk**: of course the first thing to assess is if the project is possible. For example is there oil where we want to drill? And if the project itself makes sense it is important to address the question if the project is viable. For example will the pipeline be able to carry the flow that is needed to get the required returns? While this is to a large extend an engineering problem, that only the best specialists can answer it is also a relevant risk to the Lender. If the lender would lend to an unrealistic project, then the project will fail, the SPV will go bankrupt and the loans will not be paid back. So, the Lender will have to engage specialist advisors.

- **Natural Disasters / Force Majeure**: Some events –such as a nearby supernova blast in "nearby stars", super-volcano's, comet impacts, etc.– are not possible to predict yet, have the potential to destroy the civilization as we know it and even all life on earth. Our practical knowledge on such events is limited, but from the fossil record we can tell that the probability of such event to occur is extremely low.[5] Therefore it is both impossible and unnecessary[6] to try to mitigate such risks within the framework of an infrastructure project.[7] However, there are much –smaller– events that can and should be taken into account. Natural disasters such as earthquakes, floods, etc. are exactly both probable and sizable enough so that they have to be taken into account of the design

[5] An extinction event (also known as a mass extinction or biotic crisis) is a widespread and rapid decrease in the amount of life on Earth. Such an event is identified by a sharp change in the diversity and abundance of macroscopic life. The fossil record of the last 550 million years shows every fifty to hundred million years an extinction event. In many cases they mark the transition to another era where new species dominate sea, land and air. Probably the best known extinction event is the CretaceousPaleogene extinction event (66 Ma ago) when the Chicxulub impactor ended the reign of the dinosaurs. However, the most eye-catching one is in our opinion the Holocene Extinction Event that started 10'000 years ago and is still ongoing. The reasons of the Holocene Extinction Event can be summarized in one word: humans.

[6] It is impossible to take such risks into account because we do not know exactly when the next supernova will blast (or more precisely: "has blasted" – if for example WR104 in Sagittarius has blasted 7999 years ago, –and assuming it is exactly 8'000 light-years from us– then its X-rays will wipe out most macro-biotic life next year when its X-rays destroy ozone layer and most complex molecules such as DNA.) Also it is unnecessary to take such risks into account: if indeed WR104 exploded 7999 years ago and next year all complex life will be wiped out on earth then, indeed, it would not matter that our railroad will not be finished.

[7] Indeed Extinction Events have happened in the past and will happen in the future, we cannot take them into account in risk management of our project (such as an airport, highway or a bridge), but the mathematical certainty that life will be wiped out on this planet will make an excellent LRL project for the future: build the technology and machines to spread humans or their intellect beyond this planet. This however requires a planetary effort of a planetary society with a unified humanity. Our guess is that this is at least a few generations ahead.

of the project (eg. the bridge will have to be strong enough and must not move with the resonance frequency of the wind-force such as the Tacoma Narrows Bridge in 1940[8]) and can and should be insured (both before, during and after the construction phase of the project).

- **Cost Overrun**: Every project can end up more expensive than initially planned: from a refitting of one's bathroom to the building of a nuclear power station. This can have many causes: materials used, price inflation of the materials, (un-hedged) currency exchange risk, strikes, etc. But probably the most important factors for large projects are the Technology Risk and the Completion Delay.

- **Technology Risk**: Typically in LRL the technical details of the design project are not known in advance[9] The more experienced the Sponsor is, the less risk there should be, but many complex projects need still a lot of the planning to be done when the deal is struck.[10] Also in many cases the technology might not exist or is being used for the first time (eg. building an Airport on the sea –Hong Kong– or building a 27km long Electron-Proton Colider under Geneva.)

- **Completion Delay**: As in all projects, some tasks "lie on the critical path" and will delay project delivery if not closed in time. Since large projects have so much dependencies, it is unreasonable to expect that nothing will go wrong. In many cases the large losses or reduced profitability incured is due to a delayed completion of the project (eg. the Betuweroute –as mentioned in Table 1.1 on page 115– is a project where Completion Delay caused the costs to double and the benefits to be much later available for the economy). This risk will typically end up with the Engineering company that contracts to build a certain part. The SPV for the project will typically foresee penalties for late delivery.

[8]The failure of the Tacoma Narrows Bridge in 1940 is a classical textbook example of many physics and engineering handbooks. See for example Ohanian (1994), Wikipedia (2015) or the paper Billah & Scanlan (1991)

[9]One counts on the winning consortium of Sponsors to fill in these details. This is very different from the projects where authorities are the direct and sole owners of the project, in that case they typically have a very detailed order book ready and the bidding is typically focused around pricing.

[10]There are many reasons for the limited availability of project details at the start of the projects. First of all it would be very costly and take too much time to plan all details in advance, on top of that certain aspects will only be discovered as the building starts and then plans have to be adapted anyhow. Also one can argue that project details don't have to be submitted to win the tender.

- **Credit Risk**: This form of risk is the risk that a lender bears when lending money to a borrower. The risk is obviously related to the (in)capacity of the borrower to service the loan. As in Project Finance the borrower is an SPV –with little equity compared to loans on its balance sheet– this risk becomes particularly important for the lender(s). As a mitigation for the lack of creditworthiness of the SPV, the lenders will seek claims on the construction and/or exploitation rights of the project. As the reader will understand this makes the lenders most exposed around the delivery phase, when all the credit is drawn but the security (the project) is not yet producing any cash flows.
- **Cash-Flow risk**: The risk is here that the project is not able to deliver the cash flows that were planned. This can be due to many reasons, but the most obvious are related to
  - the risk that the sales of the project's product is lower,
  - that the input materials are not available or more expensive (Feedstock Risk),
  - that input materials are more expensive because of the exchange rate (Currency and Exchange Risk),
  - that interest rates rise to that the borrower cannot service the loans appropriately (Interest Rate risk).
- **Sales Risk or Off-Taker-Risk**: This is the risk that once the product is ready that there is less demand for the product. For example electricity from windmills might substitute energy from a nuclear power plant. These changes can be a result from market dynamics related to the project (eg. transport by road and water took already the market for the new railroad because it took so long to build it), or reflect political changes (eg. a negative attitude towards air travel, nuclear power, etc.). In case that the off-taker is a large company and not the entire population the SPV will typically seek contracts of minimal take-off (eg. an oil refinery, pre-selling oil at a given price to the Tolling Company).
- **Feedstock Risk**: This is the risk that the fuel needed to run the installation that is being build becomes scarce and/or expensive. This can be related to political risk (eg. oil will become both scarce and expensive when war rages in the Middle East), pure supply limitations (eg. war destroyed part of the production capacity). If prices of the Fuel will go up, then also this will squeeze the cash-flows of the project that in their turn deteriorate the credit quality.

- **Financial Market-Risk**
  - **Currency and Exchange Risk**: This is the risk related to adverse price changes for fuel, materials, labour, etc. because of fluctuations of the exchange rate of relevant currencies. This risk can be hedged via derivatives on the relevant exchange rate. Different options are open for the investors, such as forwards (with fixed conditions) and options (which don't have to be exercised when they end out to the money). For example when one runs a project in GBP and needs to buy some materials for $ 10mln. then it is possible to get into an agreement and buy the currency today at a price that is fixed today (the forward price), with delivery and payment when the currency is needed. Alternatively, one can speculate that the dollar might decrease in value and not lock in the exchange rate today: in that case one buys a call option on Dollars with nominal $ 10mln. That option can be bought now (payment up-front) or paid "as an installment", called a SWAP agreement.
  - **Interest Rate Risk**: When the interest rate risk of a loan is not fixed, but depends on a market rate, then interest rate risk occurs for the borrower. When interest rates rise faster than his earnings, he might struggle to repay the loan. Therefore bankers will typically not propose a term loan at a fixed interest rate but rather a loan that is to be repaid at a floating interest rate and an IRS[11]. The reasons why banks would rather not package these deals were that this was beneficial for their balance sheet as the latter would be "off-balance". Obviously, this created some confusion for the buyers of such contracts.[12]
- **Political Risk**: this form of risk is related to the fact that political authority is not always very stable and that while a certain government procures the project now, by delivery of the project there might be a government that is hostile to the project. Other –related– forms of risks are the "regulatory risk". For example while the power station is being

[11] IRS stands for Interest Rate Swap. This is the derivative transaction where party A (the bank) promises to provide the difference between a floating interest rate and a fixed one. Party B (the borrower in our case) hence gets from party A the fixed interest minus the floating, the result is that party B can repay the loan on a fixed interest rate known in advance.

[12] The IRS will basically provide positive income for the buyer when interest rates rise (so that the effect of this rise is canceled out), but if interest rates increase then the value of the negative. This means that if the borrower wants to pay back his loan he still has to pay a significant amount to break the IRS.

built, the authorities might add controls, guidelines and procedures. This might result in –some parts– of the project having to be rebuilt for example, or simply more costly procedures are needed as a license to start operation seems suddenly elusive. For example in 2015 the UK government decides to stop subsidizing new onshore windmills, this will not impact existing projects too much (maybe they might even get a better price for their electricity), but it is not unknown that governments change the rules of the game while playing.

- **Model Risk**: finally all the above mentioned risks are put into a statistical model that is supposed to find out if the returns are worth the risks. Such model is never perfect and and the expected deviation of this model with the reality is called "model risk". The model can for example be over-fitted on on too many variables, or a the set of historic data is simply not representative for the future risk.
  Of course there is also another type of model involved: the models that the engineers use to calculate the amount of material used for example. Also these models are critical, if the engineers get it wrong the structure will not be strong enough.

As the reader will notice, the different forms of risk can interact, shift from one form to another and even many of the risk definitions used are overlapping or could be classified in other ways (eg. Financial Market Risk could be a combination of both Exchange Rate Risk and Interest Rate Risk) and most risks are can be summarized as "Cash Flow Risk". For example a technology risk might realize itself (eg. while building the subway in Warsaw –in 2013– many problems with water entering tunnels and movements in the sandy ground appeared. This technology risk resulted in a Time Overrun Risk, that on its turn causes that the subway will be opened later, and hence incoming cash flows will be later in the project while earlier on there were higher costs dealing with the problems underground. Almost all risks somehow will be converted into Cash Flow Risk (henceforth CFR). Getting this CFR correct is therefore crucial, but to get that correct it is essential to understand all other types of risk.

Cash Flows can be modeled with a wide range of approaches. A simple spreadsheet such as Libre-Office-Calc or Mircrosoft Excel can get you a long way in order to get a view on the big picture of the project. This "big picture" is always good to have and is not really replaced by more detailed simulations.

However to fully model the complexity of a large infrastructure projects and get a good view on the risks, a spreadsheet approach is bound to create issues. As the model gets more complex, a spreadsheet approach has the following issues:

1. **Complexity**: a spreadsheet is ideal for simple calculations, a more complex model quickly gets the level of transparency of Gordian Knot.

2. **Audit and Challenging**: because of the Gordian Knot structure of a spreadsheet it is basically only possible to unravel its inner workings for the one who designed it, seriously limiting the possibility to challenge and audit the model independently. This increases the Model Risk.

3. **Speed**: spreadsheets are multi-purpose softwares that can do a lot more than a simple calculation, but therefore carry an enormous overhead in memory use, disk space and above all this will seriously drag the speed of any calculation.[13]

4. **Technical Limitations**: while it is ideal to design a stress test (provided that the spreadsheet is structured logically[14]

5. **Limited version control and no Merging possibility**: while spreadsheets have an undo function and allow some primitive versioning, they are nowhere near to a professional (and free) versioning system such

[13]For example, in 2013 I was helping a bank in Ireland after the crash in their real estate market: roughly half of the customers had payment problems and needed would stop paying if the bank could not propose a restructured loan. The engine that they were using to calculate the best suited offer –built in Microsoft Excel– was not only sub-optimal but really slow: it took the engine 5 minutes per loan to find the best solution. I have built for them a simple application in VBA (Visual Basic for Applications), which is in itself a very slow programming language with massive overhead. The result was an engine that was a lot more user friendly, ca. 1000 times faster and more friendly for customers to keep their houses and more gentle in the use of capital for the bank (not to mention that it was possible to maintain and configure the software).

[14]We recommend a few simple rules of thumb that can be really helpful in mitigating the "Gordian Knot aspect" of a spreadsheet. The following will be helpful:

- have an organization per sheet: first sheets has all the input parameters, the second the costs, the third the income, the fourth the Profit and Loss Statement, the fifth is a dashboard with useful indicators such as NPV, IRR, etc.;
- in each sheet one should find the same time axis in the rows (example row G is always month 3 from year 2017)
- use a color coding to show what is input, calculated, etc.;
- avoid formulae and constructs that are difficult to read for humans (for example the INDEX() function)

For more explanation, see Chapter 5.2.2 on page 165.

as SVN[15] that are able to merge files to such level that it more than one programmer can edit one file and the system will find the final form of the file.

In order to make a serious Monte-Carlo simulation and/or deal with the complexity and interdependency of parameters it is advised to use a programming language in order to make the risk assessment. The market of commercial software applications is large, but there are also great applications freely available, such as the R statistical programming language[16] that has a very wide user base –and hence a lot of support and pre-defined snippets of code– and was recently bought by Microsoft.

Also the C++ on a Linux machine offers full object oriented programming capacities with a compiler that provides extremely fast binaries. Of course, there are also large amounts of commercial softwares available: SAS, SPSS, Mathematica, etc.

[15] SVN or Subversion is a brand of the Apache Foundation and can be found here: https://subversion.apache.org/

[16] For more information about R, please refer to `http://www.r-project.org/`.

Chapter 4

Risk Assessment and the Risk Matrix

Now that we have identified the major risk elements of a large infrastructure project and have touched the subject of risk allocation we are ready to try to mitigate the risks. It is customary to list the risks, mitigation, and some sort of follow up in a table and call that table the "matrix". The whole idea of a LRL project is to re-allocate risks to parties that can bear it better and if that is not sufficient to diversify risks by collaboration (called "syndication" for the lenders and "shareholder structure" for the Sponsors)

Now that all risks are listed, the next step will try to investigate which risk are worth mitigating (and if so to what level) and also which risks should be accepted. To do this one will have to compare the different risks that the company bears. One particularly useful method is called constructing a "risk matrix", it is comparable with a dashboard that tries to simplify all different risks just enough so that they can be compared and that priorities can be set.

Below we provide an example of such risk matrix.

Risk	**Mitigations in place**	**Residual* Impact**	**Prob.**	**RAG* after mit.**
		Setup Phase		

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Risk	Mitigations	Impact	Prob.	RAG
Risk Attribution	a logical setup according to the Abrahamson Principles[1], selection of right participants	2	1	G
	Construction Phase			
Feasibility	expert opinions,	2	1	G
Technology	penalties, waranties, independent engineers confirmation, retention guarantees, insurance performane policy	1	4	A
Completion Delay	expert opinions,	2	1	G
Credit Risk	CDS, credit insurance	2	1	G
CF: Model	expert opinions,	2	1	G
CF: Cost Overrun	credit support, IRS, FX options and forwards, forward supply contracts, fixed price turnkey contracts, opinions of experts on beforehand, completion guarantee from Sponsor	3	4	R
CF: Sales	Forward Take-off contracts	2	1	G
	Operation Phase			

continued on next page ...

[1]The allocation of risk had for example been studied by the following authors: Abrahamson (1973), Ashley (1977), Barnes (1983),Ward, et al. (1991), Cheung (1997). All these texts largely agree with Abrahamson who proposed the following principles. A risk is best allocated to the party

- who can willfully influence the risk by misconduct, care or behaviour;
- for whom that risks constitutes an insurable risk (note that insurance companies will use similar rules to asses if that particular risk is insurable to that party);
- who –simply– would be most efficient in carrying that risk (or it would be most efficient to allocate the risk to that party);
- who by being allocate this risk makes the project more efficient;
- who would have to cover the risk in the first instance if it would realize itself and there is no reason (following from the above stated rules) why the risk should be allocated to another party (or simply if it would be impractical to do so).

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Risk	Mitigations	Impact	Prob.	RAG
CF: Feedstock	forward supply contracts	2	1	G
CF: Sales or Off-Taker	forward contracts	2	1	G
Market: Interest Rate	IRS	2	1	G
	General Risks (Permanent Risks)			
Political	long term concession deals	2	1	G
Social (riot, civil disorder)	—	2	2	A
Natural Disaster	insurance	2	1	G
Financial: interest rates	IRS or other SWAP	1	3	G
Financial: FX rate	forward, SWAP or option	1	3	G

Table 4.1: *A hypothetical example of a risk matrix (from the Lender's point of view!). This example holds both some generic information as well as some examples: the columns marked with an asterisk (*) are sample data that could be used. Ideally one would use behind each risk factor a MCDA method to define the RAG status. Key is of course never to forget that any MCDA method that provides a full ranking discards a lot of information, no risk matrix will ever replace the work on the field and the interconnectedness with the team on the field. The abbreviations used are: CF (Cash Flow), CDS (Credit Default Swap), "after mit." (after mitigation), "prob." (probability)*

Chapter 5

Risk Quantification

In previous chapter we –quite naively– assumed that one can assess risk by looking at its probability of occurrence multiplied with its impact at occurrence. This is actually close to the approach that BASEL II proposes for assessing risk in loan portfolios. Each loan gets assigned a PD (probability of default – non-dimensional between 0 and 1), a LGD (the Loss Given Default – expressed in currency) and an EAD (Exposure at Default – non-dimensional, between 0 and 1). This allows us to express the Expected Loss (henceforth EL) as follows.

$$EL = PD \times EAD \times LGD$$

Where for the usual term loans the LGD would be the outstanding amount reduced by the value of the collateral and costs, here one would need a model to asses how much the lender would expect to get back via exploiting the asset (or selling parts, depending on the phase in which the project goes bankrupt).[1]

This simple formula helps to quantify the expected loss of a loan and is able to reflect the fact that over time the loan is paid back (so the EAD decreases), of course PD will be correlated with the time to maturity as well as EAD and LGD. While this approach might have its merits for large portfolios of small loans (large numbers), it is highly insufficient for the concentrated risk in project finance.

[1] An additional problem is that one cannot really rely in the law of large numbers as in consumer lending (unsecured lending).

The reason is that this formula largely ignores the effects of the small probabilities on large losses. This idea that the tails of the distributions can be expected to "behave well"[2]. This, of course, creates a false sense of safety. The problem is that with large and unique projects one has no idea what can go wrong, nor what its probability is nor what our exposure will be at that moment, nor how much we could still recuperate.

Simply stated, there is a small probability that it goes terribly wrong and that we loose a lot. Using the Gaussian distribution, one will simply assume that possible outcomes are very well concentrated. If that is the universe in which you live, then concept such as VaR (Value at Risk) and volatility (standard deviation) make sense. However, our economic history is littered with 5-σ-events and any return distribution shows a lot more extreme risks than that a Gaussian distribution would predict. This is risk is called "tail risk" and related to the "fat tales of the distribution".

5.1 In Search of a Risk Measure

Quintessential in the thinking about risk is that there should be an appropriate trade-off between risk and reward. When we left the trees about 6 million years ago, this was a huge risk, as we were rather slow and defenseless, but the alternatives weren't great and with hindsight we can say that this risk has been rewarded. All animals that have a brain seem to abide by this rule and humans doing large infrastructure projects are no exception. When we do something risky, then we require a suitable –potential– reward.

Return is rather easy to define: –for example for the Lender– it is simply the compound interest rate that one will charge on his loan. Risk however is more elusive. Risk is related to the potential undesirable outcomes: the cases where the Lender will not get the desired return.

Assume that we have a project that takes only one year, and has a 99.5% to yield 5%, and assume further that when the project fails that the Loss Given Default is 90%. Assume we invest \$100, then there are two possible outcomes:

1. it all works out and we get back \$105 in one year; or

2. the project fails and we get back \$10 in one year.

[2]This is largely the same as saying that the tails of the pdf converge faster to zero than x^2. On other words: the second moment must exist and hence the concept "volatility" (as standard deviation) is well defined. And –very important– on top of that the pdf is continuously decreasing towards direction of largest losses. For example a class of distributions where this works well is the class of elliptical distributions.

Is that a good project? Is the return worth the risk? That question is not so straightforward and in the following sub-sections we will propose a few measures that could potentially be used to assess risk. Note that we will cover only the essentials in the order of familiarity for most readers. A more formal presentation can be found in Appendix A on page 179.[3]

In what follows, we will denote the Return as R.

5.1.1 Variance as a Risk Measure

A most interesting paper on risk and reward is Markowitz' mean-variance criterion, published in 1952 (Markowitz 1952). He did not discuss the choice of variance as a metric for risk, but he argued that his mean-variance criterion was a much better approach than the portfolio-selection mechanisms that were in vogue at that date. These models had tried to optimize return, and entirely disregard risk. After introducing his mean-variance criterion, he concluded that this method was superior as it lead to better diversification.

With this approach, he was connecting with the line of thinking of Bernoulli (1738), who had also argued that risk-averse investors would seek to diversify: "...it is advisable to divide goods which are exposed to some small danger into several portions rather than risk them altogether" (translation from Bernoulli 1954).

This is the very foundation of project finance: it's because the projects are so large that the risk concentration would be too high for one party to bear that it is transferred to another party that will be able to bear the risk by diversifying over different project –that is where the commercial bank comes in. The bank will not invest in one project but will hold a portfolio that is diversified over sectors, content, countries, regions and size. No government will be able to get that diversification, because to start with they typically operate in one country.

However, Markowitz did not really ponder the choice of a risk measure. He simply used variance as a proxy for risk in his seminal paper. Markowitz was not the first to consider variance as a proxy for risk. Available to him were for example, Fisher (1906) who suggests variance as a measure of economic risk. Marschak (1938) additionally suggested to use mean and covariance matrices of consumption of commodities as a first-order approximation for utility.[4]

[3] Another more general approach would be to recognize that both risk and return are two functions that have to be optimized (return maximized and risk minimized). That would lead us to consider the many heuristics that are designed to solve Multi Criteria Decision Problems, and that science is called Multi Criteria Decision Analysis (henceforth MCDA)

[4] Noteworthy is that Marschak supervised Markowitz's dissertation.

Definition 5.1.1 (Standard Deviation). Let R be a real-valued random variable; its standard deviation σ is then defined as the square root of the variance:

$$\begin{aligned}
\sigma &:= \sqrt{VAR} && (5.1)\\
&:= \sqrt{E[(R - E[R])^2]} && \text{(definition of variance)}\\
&= \sqrt{E[R^2] - (E[R])^2} && (5.2)\\
&= \sqrt{\int_{\mathbb{R}} (x-\mu)^2 \, f_R(x) \, \mathrm{d}x} && (5.3)\\
& && (5.4)
\end{aligned}$$

with $\mu := E[R] = \int_{\mathbb{R}} x \, f_R(x) \, \mathrm{d}x$

A Mathematical Formulation of the MV-criterion. The following could be a simple mathematical formulation of the mean variance optimization.[5]

Suppose that we want to construct a portfolio consisting of M possible risky loans. A portfolio can be referred to via the weights over the different loans:

$$\boldsymbol{w} = (w_1, w_2, \ldots, w_M)'$$

with condition

$$\sum_{i=1}^{M} w_i = 1 \tag{5.5}$$

The returns[6] of the possible loans $\boldsymbol{R} = (R_1, R_2, \ldots, R_M)'$, have expected values $E[\boldsymbol{R}] =: \boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_M)'$, and have an expected covariance matrix defined by:

[5] Note that this part does not follow Markowitz's formulation, which was rather geometrical and limited to 3 possible loans. This formulation is more general, but uses exactly the same principles.

[6] We direct the reader to Appendix **??** on page ?? for a precise definition of return. While a risk-reward optimization is possible for all the proposed definitions of return as "reward", we suggest for this purpose to think of return as the percentage of increase in the value of an investment.

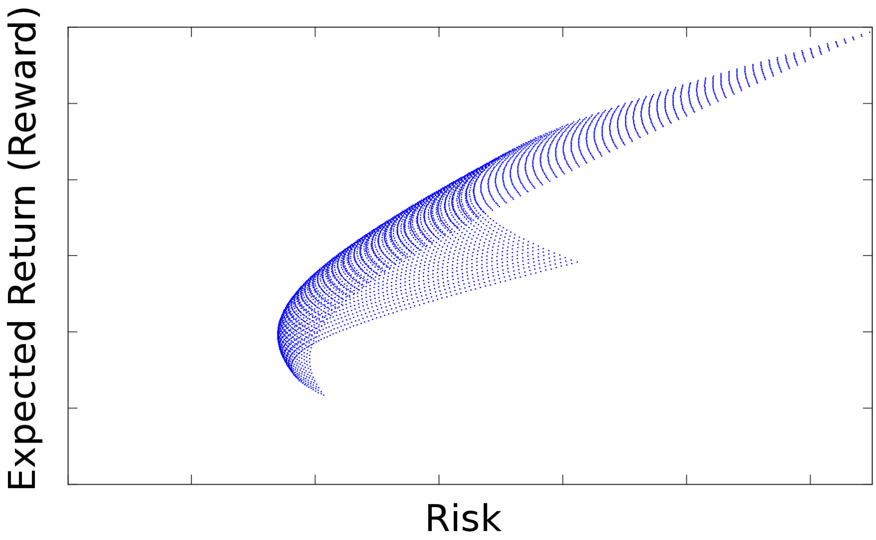

Figure 5.1: *This figure shows possible portfolios that consist of three loans, when no short selling is allowed. The plotted portfolios differ by 1% steps in composition. In general the portfolios cover a surface in the (σ, R)-plane. One will notice that the portfolios with the lowest variance include all loans. Or in other words, adding any asset that is not 100% correlated allows us to reduce volatility for a fixed return. — after De Brouwer (2012)*

Definition 5.1.2 (Covariance Matrix).

$$\mathbf{\Sigma} := \begin{pmatrix} E\left[(R_1-\mu_1)(R_1-\mu_1)\right] & \cdots & E\left[(R_1-\mu_1)(R_1-\mu_M)\right] \\ \vdots & & \vdots \\ E\left[(R_M-\mu_M)(R_1-\mu_1)\right] & \cdots & E\left[(R_M-\mu_M)(R_M-\mu_M)\right] \end{pmatrix}$$
$$=: \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1M} \\ \vdots & & \vdots \\ \sigma_{M1} & \cdots & \sigma_{MM} \end{pmatrix}$$

Where σ_{ij} is the covariance between asset i and asset j.

Note that $\sigma_{ii} = \sigma_i^2$, and that in general $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$. With ρ the correlation coefficient.

Using this formulation, it is easy to see that the return, expected return, and expected variance of a portfolio p are respectively defined by:

$$R_p = \boldsymbol{w}'.\boldsymbol{R} \tag{5.6}$$
$$\mu_p = \boldsymbol{w}'.\boldsymbol{\mu} \tag{5.7}$$
$$\sigma_p^2 = \boldsymbol{w}'.\mathbf{\Sigma}.\boldsymbol{w} \tag{5.8}$$

The mean variance criterion as developed by Markowitz is now reduced to

$$\begin{cases} \min_{\boldsymbol{w}}\{\boldsymbol{w}'.\mathbf{\Sigma}.\boldsymbol{w}\} \\ \max_{\boldsymbol{w}}\{\boldsymbol{w}'.\boldsymbol{\mu}\} \end{cases} \tag{5.9}$$

with constraint:

$$\boldsymbol{w}'.\boldsymbol{\imath} = 1 \tag{5.10}$$

with: $\boldsymbol{\imath}' = (1, 1, \ldots, 1)$

However 5.9 is a problem that leads to an infinite set of solutions which cannot be ordered by these two criteria: all the portfolios that lie on the "efficient frontier" are solutions to this problem.

We will therefore choose one:

$$\mu_0 = \boldsymbol{w}'.\boldsymbol{\mu}$$

Hence we replace 5.9 with constraint 5.10 by:

$$\min_{\boldsymbol{w}}\{\boldsymbol{w}'.\boldsymbol{\Sigma}.\boldsymbol{w}\} \tag{5.11}$$

with constraints:

$$\begin{cases} \mu_0 = \boldsymbol{w}'.\boldsymbol{\mu} \\ \boldsymbol{w}'.\boldsymbol{\imath} = 1 \end{cases} \tag{5.12}$$

This formulation of the mean variance criterion is generally referred to as "the risk minimization formulation". It is a quadratic optimization problem with equality restraints and is hence solved by using, e.g. the method of Lagrange multipliers.

Its solution is given by:

$$\boldsymbol{w} = \frac{1}{a\,c - b^2}\boldsymbol{\Sigma}^{-1}\left(c\boldsymbol{\imath} - b\boldsymbol{\mu} + (a\boldsymbol{\mu} - b\boldsymbol{\imath})\mu_0\right)$$

where we assume that the covariance matrix, $\boldsymbol{\Sigma}$ is positive definite, and where the scalars a, b, and c are defined by:

$$\begin{aligned} a &:= \boldsymbol{\imath}'.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\imath} \\ b &:= \boldsymbol{\imath}'.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} \\ c &:= \boldsymbol{\mu}'.\boldsymbol{\Sigma}^{-1}.\boldsymbol{\mu} \end{aligned}$$

Of course, one can also choose alternative formulations of the mean variance hypothesis, such as the "expected return maximization formulation" or even the "risk aversion formulation".

A most interesting and theoretically very satisfying corollary to this is that the volatility aversion of the investor can be expressed by a simple utility function, where one parameter, d, characterizes the variance aversion.

$$U_{MPT} = \mu - \frac{\sigma^2}{d} \tag{5.13}$$

Variations to the Variance-Theme . Markowitz himself mentioned in his Nobel Lecture (Markowitz 1991) that "The basic concepts of portfolio theory came to me one afternoon in the library while reading John Burr Williams' *The Theory of Investment Value*" (Williams 1938). However Williams believed that all risk could be diversified away: "[talking about bonds] ...Given the adequate diversification, gains on such purchases will offset losses, and a return at the pure interest rate will be obtained. This the *net risk* turns out to be nil" (pp. 67–69).

To Markowitz we owe the mathematical formulation of the idea of diversification of investments. He also made clear that not all risk can be diversified away, and that there is always a residual risk—the systematic risk. He also pointed out that it is not the variance of the individual investment that is the most important, but its contribution to the risk of the portfolio. See Equation 5.8 on page 142, and more explicitly (expanding the matrix notations):

$$\begin{aligned}\sigma_p^2 &= \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} w_i w_j \sigma_{ij} \\ &= \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} w_i w_j \rho_{ij} \sigma_i \sigma_j\end{aligned}$$

This formula makes clear that it might make sense to add a risky loan to the portfolio under the condition that –due to correlation effect– the overall risk of the portfolio decreases.

Seven years after his seminal paper in (1952), Markowitz introduced an alternative to variance (or standard deviation) as risk measure, in his book "Portfolio Selection, Efficient Diversification of Investments" (Markowitz 1959). This alternative was the semi-variance, a measure that would only take into account adverse movements.

Definition 5.1.3 (Semi-Variance).

$$S := E\left[\left(R^-\right)^2\right]$$

with $R^- := \begin{cases} R & \text{if } R \leq 0 \\ 0 & \text{if } R > 0 \end{cases}$

Later, in his Nobel Lecture, 7 December 1990, (printed in Markowitz 1991), Markovitz defines semi-variance slightly differently:

Definition 5.1.4 (Semi-Variance).

$$S := E\left[\min(0, R - c)^2\right]$$

thus explicitly introducing a parameter c which could hold the information of a non-zero investment target. In his Nobel Lecture he mentions that he "proposes semi-variance" and that "semi-variance looks more plausible than variance".

He further comments on his choice of variance as a risk measure:

> "Variance is superior with respect to cost, convenience, and familiarity. For example, roughly two to four times as much computing time is required (on a high speed electronic computer) to derive efficient sets based on S than is required to derive efficient sets based on V [variance].
> (...)
> This superiority of variance with respect to cost, convenience, and familiarity does not preclude the use of semi-variance.
> (...)
> Analyses based on S tend to produce better portfolios than those based on V.
> (...)
> The proper procedure, it seems to me, is to start with analyses based on variance. Analyses based on semi-variance, and those based on still other criteria."
> – (Markowitz 1959) - p. 193 and 194.

This makes clear that not even Markowitz himself puts variance forward as a choice in its own right. His innovation lies in how he proposes a two-parameter model to optimize portfolios and thereby creates a framework that has become the foundation for many other portfolio theories.

Whereas in the 1950s, the speed of an "electronic computer" was definitely an issue, in our times this is certainly a lesser concern. After the global meltdown of 2008, convenience and familiarity are also no longer relevant arguments for risk measure. As we will see in the next chapter, Value at Risk gained sufficient popularity to be commonly used as a risk measure on banks. The fact that this did not prevent the crisis, together with the arguments about sub-additivity (put forward in Chapter 5.1.2 on the facing page), will encourage us to look to alternatives.

This does not exclude the possibility that mean-variance optimization is an interesting method that yields reasonable results in many cases. It has been the basis of many developments, such as CAPM and the Black-Litterman model.

Roy (1952) independently found the similar equations as Markowitz, and added a deeper formulation that also makes sense for non-ellipical distributions and provided important guidance on which portfolio to select—while Markowitz left it up to the investor to define a maximum variance or minimal return.[7] Roy suggested selecting the portfolio that maximizes $\frac{\mu_p - d}{\sigma_p^2}$, where d is the disaster level or subsistence level.[8] – for more background, see De Brouwer (2012), p. 45—48.

In his paper, "Markowitz's 'portfolio selection': A fifty-year retrospective", Rubinstein (2002) reports that "Markowitz's approach is now commonplace among institutional portfolio managers who use it both to structure their portfolios and measure their performance. (. . .) and is even being used to manage portfolios of ordinary investors". He concludes his paper with, "Markowitz can boast that he found the field of finance awash in the imprecision of English and left it with the scientific precision and insight made possible only by mathematics".

However the essential problem remains: variance might be related to risk under certain conditions (such as when returns follow a Gaussian distribution), and in large infrastructure projects that is clearly not the case. Typically projects fail and incure an enormous loss, get delayed and need additional financing or simply work out and produce. So the distribution of returns will be anything but Gaussian. So we need to dig deeper and find a risk measure that is related to those extreme losses (the tail of the distribution).

[7]In his Nobel Lecture, Markovitz mentions that also Roy has a rightfull claim to have solved the asset allocation puzzle, and he conjectures that only he got the Nobel prize because people tend to use his method more.

[8]Markowitz received the Nobel Prize in 1991 mainly for this contribution. In (Markowitz 1999), he writes "Comparing the two articles, one might wonder why I got a Nobel Prize for mine and Roy did not for his. Perhaps the reason had to do with the differences described in the preceding paragraph, but the more likely reason was visibility to the Nobel Committee in 1990", and admits that "On the basis of Markowitz (1952), I am often called the father of Modern Portfolio Theory (MPT), but Roy (1952) can claim an equal share of this honor".

5.1.2 Value at Risk (VaR)

Value at Risk (henceforth VaR) appears naturally when searching for a risk measure that is asymmetric (is a "downside risk measure"), relative to a target, and easy to understand. Actually Roy (1952)'s formulation of his "Safety First Criterion" leads directly to optimizing VaR. Further, Value at Risk satisfies many of the criteria that Rachev, et al. (2008) proposed as "desirable properties of a risk measure".

Before defining the concept of Value at Risk, it is useful to introduce the basics of quantiles, because VaR is essentially a quantile.

Definition 5.1.5 (Quantile). $q_{(\alpha)}$ is called the α-quantile of a random variable X, if and only if

$$P[X < q_{(\alpha)}] \leq \alpha \leq P[X \leq q_{(\alpha)}]$$

The quantile exists only at points where the cumulative distribution function is continuous, however it is always possible to define a quantile function as follows.

Definition 5.1.6 (Quantile Function). Let F_X be the cumulative distribution function, and F_X^{-1} its inverse, then the quantile function is defined as

$$Q_X(\alpha) := F_X^{-1}(\alpha) = inf\{x \in \mathbb{R} : \alpha \leq F_X(x)\}$$

With $F(x)$ the cumulative distribution function, and F^{-1} the inverse cumulative distribution function. The Value at Risk (VaR) is now easy to define.

Definition 5.1.7 (Value-at-Risk (VaR)). For the stochastic profit variable, absolute return $\mathcal{P}$, and a probability $\alpha \in [0, 1]$, we define the Value at Risk (VaR):

$$VaR_\alpha(\mathcal{P}) := -Q_\mathcal{P}(\alpha)$$

Note. The minus sign in the definition is introduced in order to make the definition semantically coherent. A Value at Risk of 100€ corresponds to loss of 100€. A negative VaR corresponds to a situation in which profit can be expected with a probability of at least $(1-\alpha)$.
If we were to use a loss variable $\mathcal{L} := -\mathcal{P}$, then we would not need the minus sign in the definition of VaR any more. Indeed, it is possible to characterize VaR also in terms of that loss variable $\mathcal{L}$:

$$\begin{aligned} VaR_\alpha(\mathcal{P}) &= Q_\mathcal{P}(\alpha) \\ &= Q_{-\mathcal{L}}(\alpha) \\ &= Q_\mathcal{L}(1-\alpha) \end{aligned}$$

Similarly, VaR can also be characterized via the wealth variable, V, that represents the amount lent to the SPV.

$$\begin{aligned} VaR_\alpha(\mathcal{P}) &= VaR_\alpha(V - V_0) \\ &= VaR_\alpha(V) - V_0 \end{aligned}$$

In this case, a Value at Risk of 100€ corresponds a shortfall of 100€ relative to the investment target V_τ.

Many authors use different definitions, but this way has the advantage of being intuitive. This is important when the method will be used in real-life situations where the investor is generally not knowledgeable about these concepts.

Another advantage is that this definition implies that a higher Value-at-Risk corresponds to a higher risk.[9] This is also the stance that is taken in the

[9] This way of defining risk is used, for example, by (Acerbi, et al. 2001), and is well-adapted to use in portfolio theory, where practitioners generally use returns and values, and are interested in the left tail of the distribution. If, for example, one considers an insurance portfolio as "the aggregated value of the claims", then it is advisable not to use the minus sign.

definitions of coherent risk measures—see Appendix A on page 179.

VaR can also be used as a criterion that seems to be most suitable to optimize portfolios. There are two ways of attempting this:

- A first method is to select an optimal portfolio in the (VaR, R) plane, and thus basically replace standard deviation by VaR in the model of Markowitz. This method assumes that VaR is a valid and good risk measure, and that a trade-off between VaR and return will lead to an optimal portfolio (which can then be found by a utility function $U = U(VaR, R)$.
- There is also the possibility of selecting from the (σ, R) optimal portfolios those which have the lowest VaR. The idea behind this method is in essence assuming that the Markowitz method correctly uses standard deviation as a measure of risk, and that the utility is related to VaR. This is closely related to the approach of Wilson James (1997). This is a very promising approach that indeed tries to select an efficient portfolio, so that the probability of achieving the investment goal is maximized.

VaR is a risk metric that is related to the probability of losing, but it is actually the answer to a rather strange questions: "what is the lowest amount that I can lose given a confidence level of $(1 - \alpha)$?". Rational people woud rather be interested in a maximum amount (or average) that can be lost and they will also realize that it totally disregards the tail risk.

However there is another (intimately related), but far more worrying, problem. Let's consider the following examples:[10]

Example 5.1.1. Let bank A have one limited recourse loan of 100€. Assume further that the project has a 0.007 probability of defaulting over a time horizon of one year. Then $VaR_{0.01} = 0€$ for the loan over one year.

Example 5.1.2. Let bank B have two limited recourse loans of 50€, and assume that the default probabilities of both loans are independently distributed, and that each loan has the same probability of default over one year (0.007). Then $VaR_{0.01} = 50€$ for the two bonds over one year.

This shows that there is something very fundamentally wrong with VaR: when we diversified our exposure over two loans in stead of concentrating it on one the risk has to go down, but according to VaR it went up. This

[10]In these example we use the viewpoint of the Lender, however it is trivial to take the viewpoint –for example– of the Sponsor: simply replace "loan" with "equity investment" and "bank" by "company", etc.

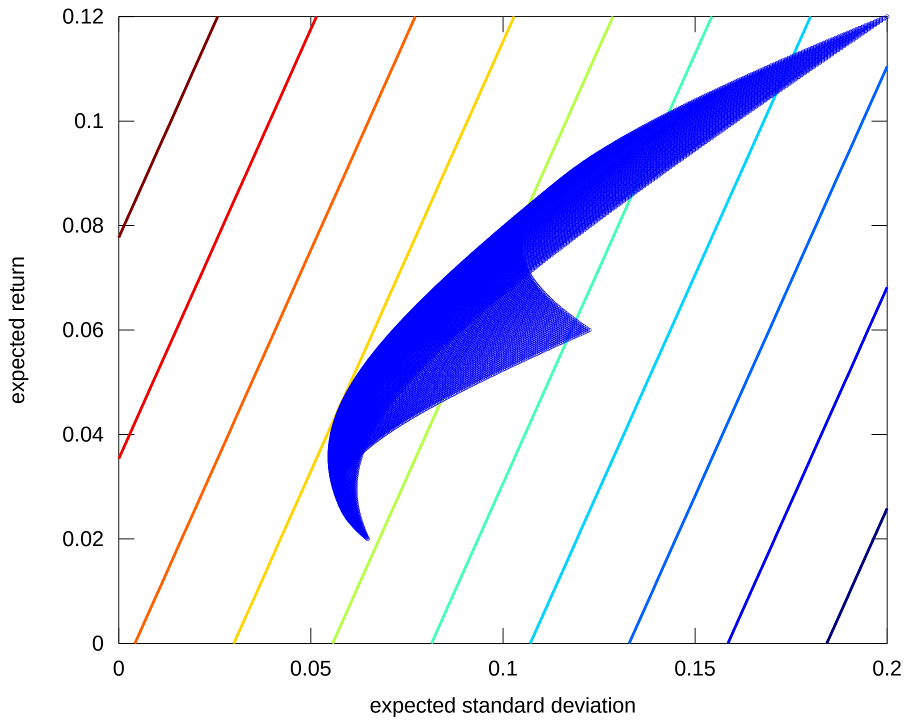

Figure 5.2: *For a return that follows a Gaussian distribution a portfolio that is optimal for VaR is also optimal for VAR and σ. This graph illustrates the evolution of VaR in the (σ, R) plane for normally distributed assets: it shows the set of acceptable portfolios (the shaded area) and adds the iso-VaR lines under the assumptions of no short selling and multivariate normality). The lowest VaR is in the lower right corner, the highest in the upper left.*
*The iso-VaR lines are drawn for a confidence level of 99%, and the values can be thought of as data for three assets (cash, bonds, and equities), as defined in Appendix **??** on page ??; and with an investment horizon $T = 1\ year$. Note that under the assumption of elliptic distributions, VaR will be convex.*

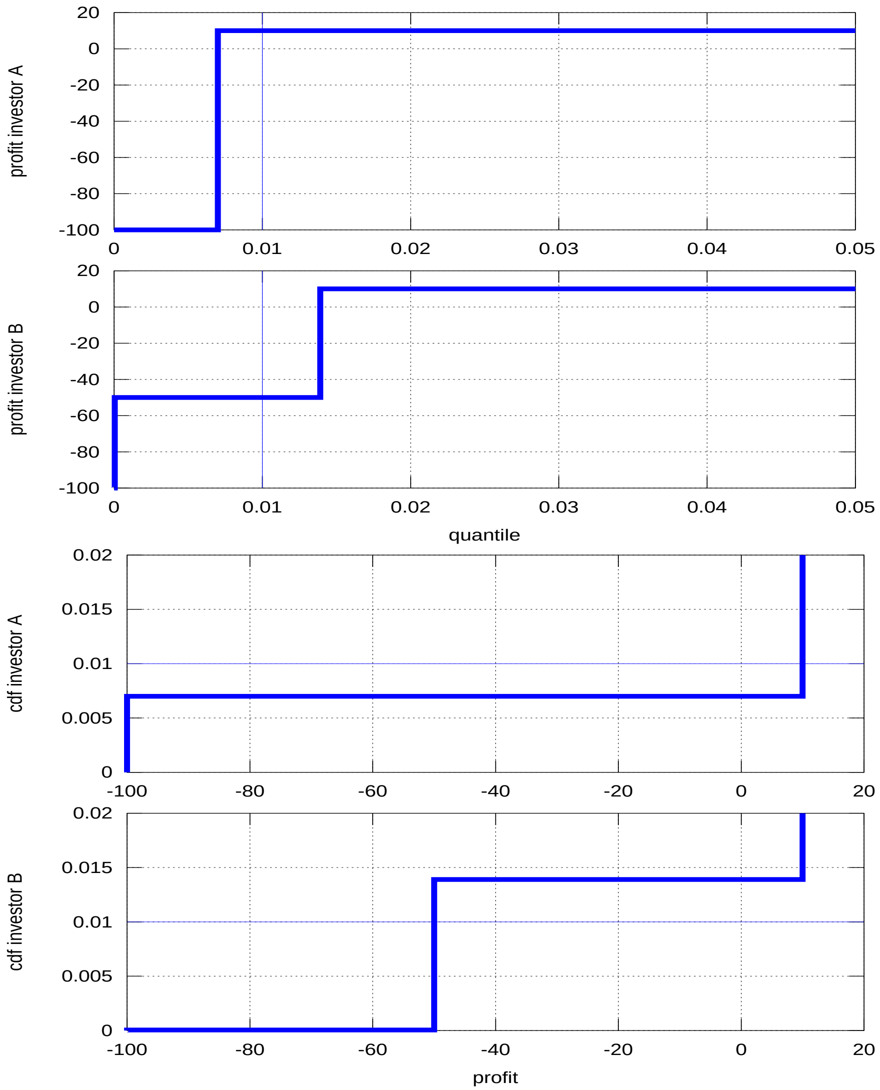

Figure 5.3: *This figure shows the quantile function for Examples 5.1.1 and 5.1.2 in the two top graphs; and the cumulative distribution function (cdf) in the two bottom graphs. The probability level of* 0.01 *returns a zero VaR in the non-diversified case (investor A), whereas in the diversified case (investor B) the VaR is 50€. Note that even investor B has a probability of* 0.000049 *of losing 100€.*

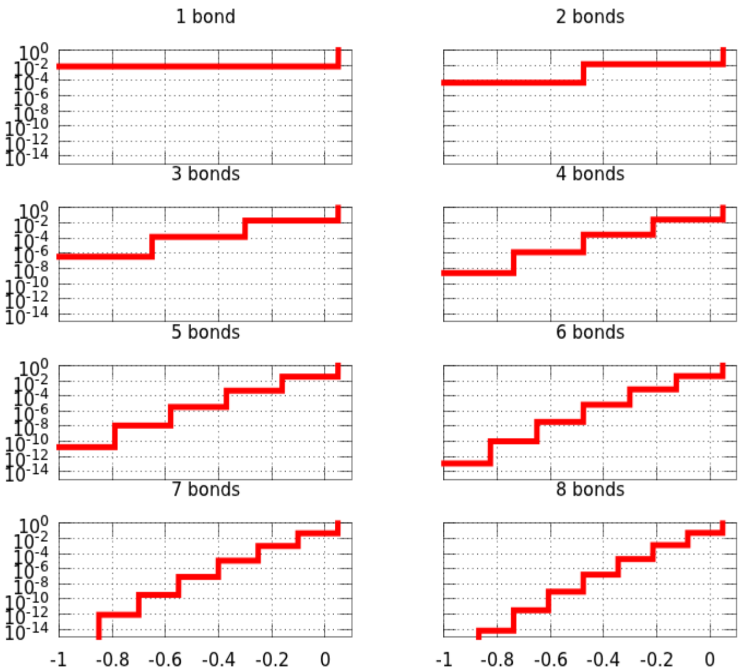

Figure 5.4: *The six graphs show the cdf for increasing numbers of bonds in portfolios—as defined in 5.1.1 and 5.1.2. With this presentation, we can see what happens with the VaR: it suffices to study the behaviour of the* 0.01*-quantile. This quantile corresponds to a value of the cdf of* 0.01 *(Note that the scale on the* y*-axis is logarithmic, and that the value of interest will be* 10^{-1}*). We notice that for one bond, the* 0.01*-quantile (and hence VaR) equal* -5%*. In the second graph, it is clear that the VaR is* -47.5%*. Increasing the number of bonds further, we notice that the VaR will decrease slowly, but that when another step passes the* 0.01 *mark on the* y*-axis, then VaR will make a jump upwards!*

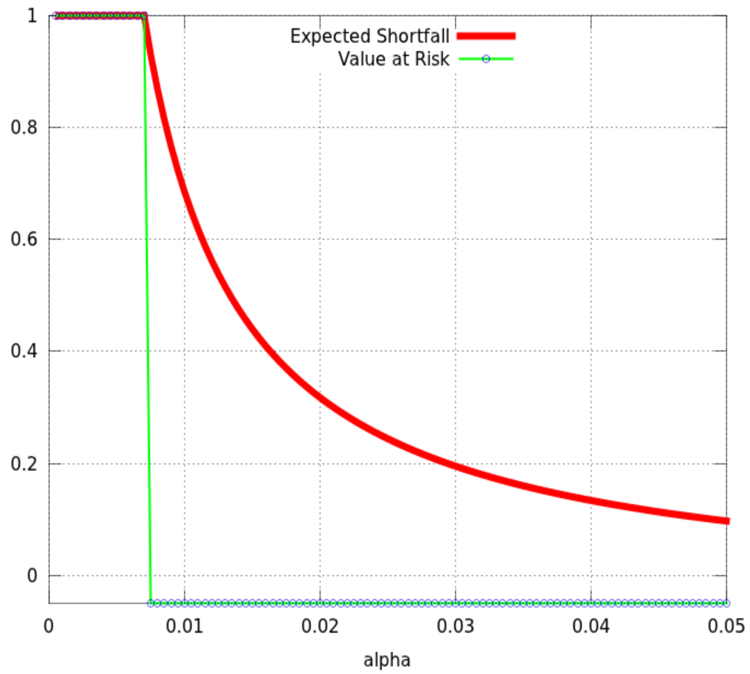

Figure 5.5: *ES_α and VaR_α are shown as a function of α, for the case where we have one bond—defined in 5.1.1. If one believes that VaR is a reasonable risk measure, then one must accept the fact that the risk drops from 100% to -5% when the confidence interval passes from $(1-0.006999999)$ to $(1-0.007)$. This is a strong argument for stating that VaR is not internally coherent. It is very reassuring that ES is continuous in α, and that hence small variations in confidence level will result in small variations in risk.*

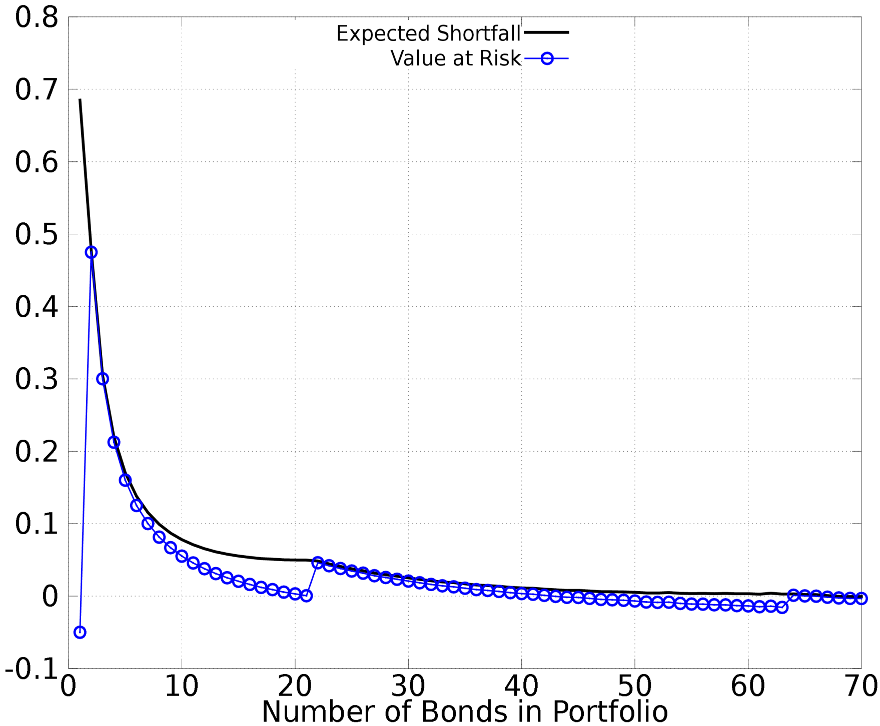

Figure 5.6: *$ES_{0.01}$ and $VaR_{0.01}$ are shown as a function of the number of bonds in the portfolio, as defined in 5.1.1 and 5.1.2. Note that VaR displays a false minimum for* 1, 21 *and* 64 *bonds.*

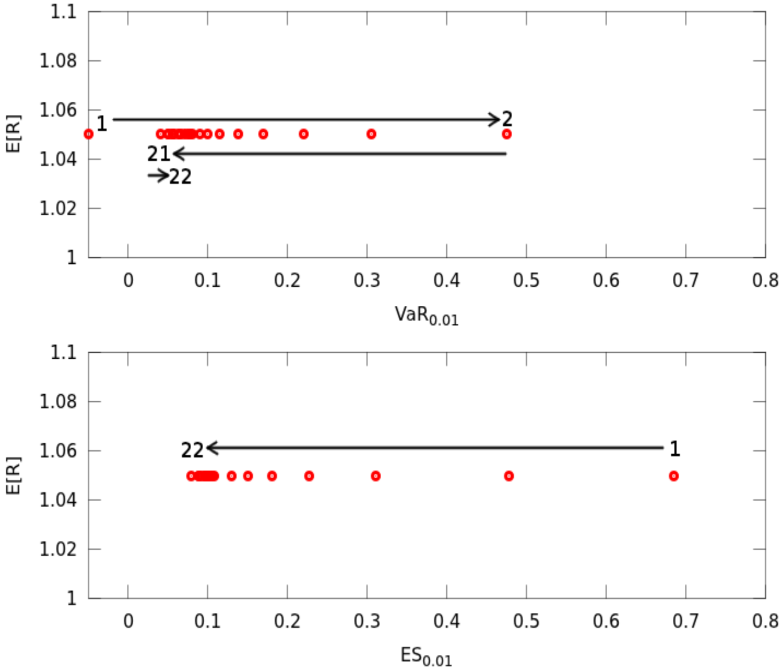

Figure 5.7: *The efficient frontier for $ES_{0.01}$ and $VaR_{0.01}$ are shown as a function of the number of bonds in the portfolio, as defined in 5.1.1 and 5.1.2. (The numbers on the plot are the numbers of bonds in the portfolio). Note that VaR displays a false minimum for* 1 *bond and for* 21 *bonds. ES, on the contrary, will systematically decrease as diversification increases; VaR behaves erratically as a function of diversification.*
This is, of course, another view of the previous graph (Figure 5.6 on the preceding page).

means that VaR can counteract diversification[11] Mathematicians would call this "VaR is not sub-additive" (see De Brouwer (2012) and De Brouwer (2014) for example) or "not convex". This actually means that in any logical and coherent definition of "risk" VaR is not to be considered as a risk measure. It is a quantile and in a parallel universe where risks follow a Gaussian distribution, one can indeed use it as a risk measure for those risks, but that is not our universe and not project finance.

Many other problems with VaR have been reported:

- Daníelsson (2002) finds that "for regulatory use, VaR may give misleading information about risk, and in some cases may actually increase both idiosyncratic and systemic risk".
- Another aspect is that convexity of the risk surface is a consequence of sub-additivity and positive homogeneity (see Acerbi et al. 2001).
- Yamai & Yoshiba (2002b) find empirical evidence that using VaR in practice as a risk measure can be dangerous, because of the fact that VaR is not sub-additive and does neglect the form of the distributions beyond the VaR level. However, they also stress that Expected Shortfall (the coherent alternative to VaR) is sensitive to the stability of the parameters of the distribution of the tail risk.
- The same authors use simulation-based multivariate extreme value distributions (Yamai & Yoshiba 2002a) to model markets under stress, and find that both VaR and ES may underestimate the tail risk, and that "ES has less of a problem in disregarding the fat tail and the tail dependence than VaR does".

5.1.3 Expected Shortfall (ES)

In Search of a Simple Coherent Risk Measure

In their seminal paper "Thinking Coherently" in (1997) Artzner, et al. proposed a set of axioms that a risk measure should comply with. It was compelling and intuitive, but there followed an impasse of a few years because there was no known risk measure that could practically be used and that would be coherent. The water between practitioners (bankers, asset managers, insurance

[11] This is one of the reasons why focus on VaR (as in Basel II for example) can be very dangerous. Lehman Brothers was always reporting good VaR numbers, but went bankrupt. VaR in essence ignores the tail of the distribution, discards all the information about the worst outcomes: that is not what a risk measure should do.

companies, and regulators) and academics was suddenly very deep. Virtually all practitioners adopted VaR, probably because of its simplicity, wide applicability to all different kinds of assets and liabilities, and its being sufficiently universal to be applied to many different risks and asssets on a bank's balance sheet. However, the afore mentioned paper had put forward four axioms that were well in line with what one would expect from a risk measure; yet VaR did not fulfil these axioms, and hence was not a coherent risk measure.

Because VaR violates the axioms of coherence, it can and will produce incoherent results. VaR calculations can result in lower VaR numbers for more risky portfolios, as has been shown in Example 5.1.1 on page 149 and Example 5.1.2 on page 149. VaR could lead to situations where people believe they have risk under control, but they are actually building a time bomb. Nonetheless, for a few years there was no alternative, until Pflug in (2000) proved that Expected Shortfall fulfilled all the criteria of coherence.

it appears that in order to get a coherent risk measure, one needs to answer a closely related question: "What is the *expected* loss in the $100\alpha\%$ worst cases?".[12] The answer to that question leads to the concept known as Expected Shortfall.[13]

Characterisation for ES for Discrete Distributions

By using the average of the $100\alpha\%$ lowest results of a number of realizations $\{\mathcal{P}_i\}_{\{i=1,\ldots,N\}}$ of the stochastic variable $\mathcal{P}$, one has finally a coherent risk measure, known as expected shortfall (henceforth ES) or conditional value at risk.

Let us denote the sorted values of the n-tuple $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_N)$ as $(\mathcal{P}_{1:N}, \mathcal{P}_{2:N}, \ldots, \mathcal{P}_{N:N})$; and let us approximate the number of the $100\alpha\%$ lowest results by

$$n = max\{j|j \leq \alpha N, j \in \mathbb{N}\} = int(N\alpha)$$

Remark 5.1.1. Any other form of rounding or truncating of $N\alpha$ is acceptable, and would also lead to (another) coherent risk measure.

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[12]One could even argue that the minimal loss in the $100\alpha\%$ worst cases is not the most relevant information; the worst loss or the expected loss provide much more relevant information for the risk-averse investor.

[13]In the literature, Expected Shortfall (ES) is also referred to as CVaR (Conditional Value at Risk), Average Shortfall, AVaR (Average VaR), ETL (Expected Tail Risk); additionally, for continuous distributions, TCE (Tail Conditional Expectation) is the same. For mor deeper information and proof we refer to (De Brouwer 2012).

Hence we can use the following natural estimator for $q_{(\alpha)N}(\mathcal{P})$

$$q_N(\alpha;\mathcal{P}) = \mathcal{P}_{n:N} \tag{5.14}$$

Then the natural estimator for the expected loss in the $100\alpha\%$ (or the $100\alpha\%$ Expected Shortfall) worst cases is given by:

Definition 5.1.8 (Expected Shortfall (ES)). Let $\mathcal{P}_{i:N}$ be the observations of stochastic variable $\mathcal{P}$ ordered from low to high and, n the number of lowest observations that correspond to the $100\alpha\%$ lowest observations, $\mathcal{P}_{n:N}$ be the estimator for $q_N(\alpha;\mathcal{P})$, then the Expected Shortfall is the expected loss in the $100\alpha\%$ worst cases of the stochastic variable $\mathcal{P}$ representing the absolute returns.

$$\begin{aligned} ES_{(\alpha)N}(\mathcal{P}) &:= -\frac{\sum_{i=1}^{n} \mathcal{P}_{i:N}}{n} \\ &= \text{- (average of the worst } 100\alpha\% \text{ realizations)} \end{aligned}$$

For the technical details we refer to De Brouwer (2012) (pages 122 to 128), but the definition A.2.3 can be rewritten with the aid of left-generalized inverse function $F_{\mathcal{P}}^{\leftarrow}$ of the cumulative distribution function $F_{\mathcal{P}}(x)$, which is defined as;

$$F_{\mathcal{P}}^{\leftarrow}(p) := inf\{p|F_{\mathcal{P}}(p) \geq p\}. \tag{5.15}$$

We note that for continuous distributions $F_{\mathcal{P}}^{\leftarrow}(p) = Q_{\mathcal{P}}(p)$ and we are ready to rewrite the characterization of ES.

Definition 5.1.9 (Characterisation of ES for continuous profits). Let $\mathcal{P}$ be a continuous stochastic profit variable –in nominal terms– with left-generalized inverse function $F_{\mathcal{P}}^{\leftarrow}$, quantile function $Q_{\mathcal{P}}(p)$, and probability density function $f_{\mathcal{P}}$, and $\alpha \in [0, 1]$, then the α expected shortfall

$(ES_{(\alpha)}(\mathcal{P}))$ can also be characterized by

$$ES_{(\alpha)}(\mathcal{P}) = -\frac{1}{\alpha}\int_0^\alpha F_{\mathcal{P}}^{\leftarrow}(p)\,\mathrm{d}p \quad (5.16)$$

$$= -\frac{1}{\alpha}\int_0^\alpha Q(p)\,\mathrm{d}p \quad (5.17)$$

$$= -\frac{1}{\alpha}\int_0^\alpha VaR_{(\alpha)}(\mathcal{P})(p)\,\mathrm{d}p \quad (5.18)$$

$$= -\frac{1}{\alpha}\int_{-\infty}^{Q_{\mathcal{P}}(\alpha)} f_{\mathcal{P}}(\mathfrak{p})\,\mathrm{d}\mathfrak{p} \quad (5.19)$$

The equivalence with Definition 5.19 is intuitively understandable; and Equation 5.18 illustrates the alternative name "average VaR" or "Conditional VaR".

This characterization makes immediately clear that ES is continuous in α. This is a very important property, since otherwise the risk measure of α and $\alpha + \Delta$ can be very different, even for a very small Δ. This immediately illustrates how one of the problems with VaR is solved by ES.

Acerbi et al. (2001) conclude that ES has the following advantages: ES is

- *universal*: it can be applied to any financial instrument and to any form of underlying risk,
- *complete*: it can produce a unique global assessment that aggregates different forms of risk,
- *simple*: it is the answer to a very natural question and has a clear interpretation.

Note. The formulae in the previous example are also valid for other definitions of return. Indeed, one can also define the expected shortfall as a percentage via $ES(R)$. However, note that this will not lead to a coherent risk measure. For example, $ES(R)$ would not be positively homogeneous. A solution might be to use $V_0 ES(R)$, which is essentially the same as $\mathcal{P}$.

Note. A natural way to extend the concept of ES for $\alpha \to 0$ is to use the worst-case outcome for X:

$$ES_{(0)} := -ess.inf\{X\}$$

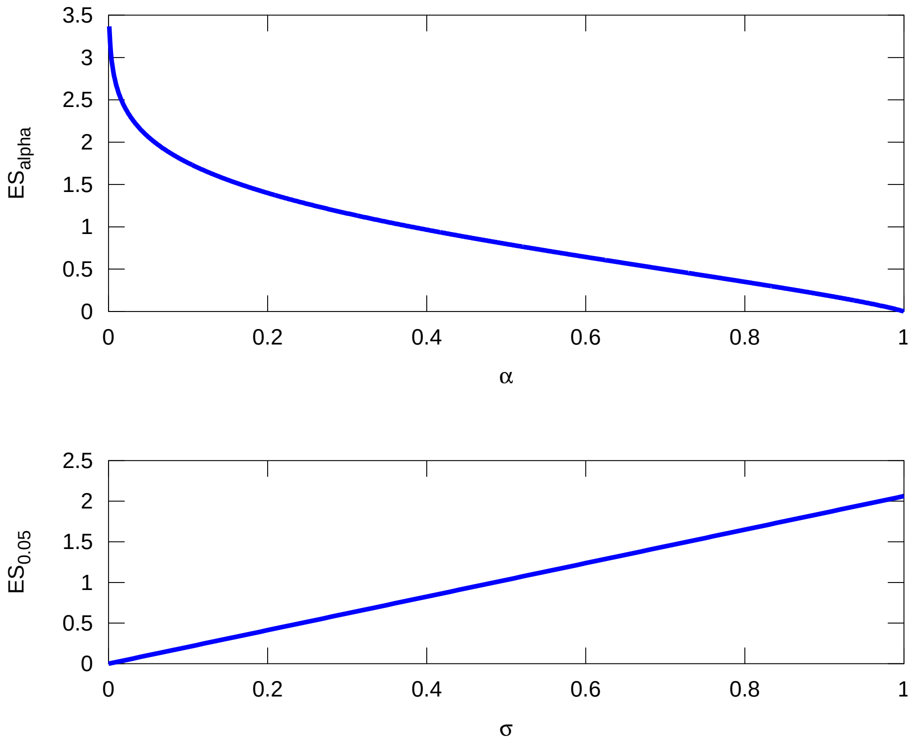

Figure 5.8: *This figure shows how ES behaves as a function of α (upper graph) and as a function of σ (lower graph) for the Gaussian distribution: ES is linear in σ. Hence for the normal distribution, the same portfolios will be efficient for both mean-ES optimization as for mean-variance optimization.*

where $ess.inf\{X\}$ is the essential infimum[14].

ES is a coherent risk measure

For a formal proof that ES is a coherent risk measure we refer to Pflug (2000), Acerbi & Tasche (2002a) or De Brouwer (2012).[16] In this book we will limit ourselves discussing the impact and advantages of using a coherent risk measure.

In Figure 5.9 on the next page we illustrate intuitively the critical difference between VaR (no "risk measure", or at least not a "coherent" risk measure) and ES (a coherent risk measure). The plots show to hypothetical probability density functions (pdfs): on the left side one that follows a Gaussian distribution and on the right side one that does not follow a Gaussian at all. The distribution on the right side is similar to a Gaussian above a certain level, but the extreme risks (tail-risk) is very different: in stead of being very confined –as so typical for a Gaussian pdf– one will notice that losses of -300 are possible (on the left this is rather excluded, as the Gaussian pdf converges very fast to zero).

In that example VaR does not see a difference! However, ES will find an important difference. The standard deviation, will also increase and –at least– indicated that both pdfs are not the same.[17]

[14]The essential infimum is defined as

$$ess.inf(X) := sup\{a \in \mathbb{R} : \mu\left(\{x : f(x) > a\}\right) = 0\}$$

and is thereby an extension of the concept of infimum,[15] in the sense that if the infimum does not exist, the essential infimum may still exist, but it disregards the values of the set with measure zero. In other words, almost everywhere the essential infimum is the infimum, except on a subset with measure zero.

[16]Further, via the theory of "spectral risk measures" or "distortion risk measures" (both explained in De Brouwer (2012)) one can show that ES is actually the most simple coherent risk measure.

[17]To illustrate how absurd VAR (or σ) would be as a risk measure, it is sufficient to use distribution functions that are exactly the same: for example a normal distribution function with average £ 100 and standard deviation £ 2 for project A, and in project B everything is the same, except for the condition that the Sponsor will take first £ 3 out of the SPV before the SPV pays the bank. In that case the return of the project keeps the same distribution, but simply is moved to the left, seriously increasing the probability that the bank will not get her investment back. In this case ES and VaR (most probably) will see a difference, but VAR will fail.

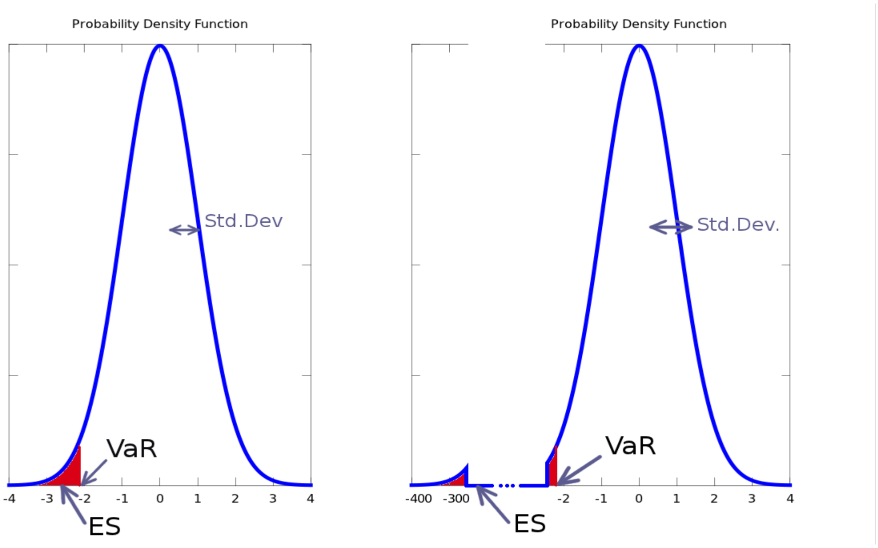

Figure 5.9: *In this figure we show left a confined distribution (eg. a Gaussian distribution), on the right a distribution that is largely the same, except for the extreme risks. In both cases a typical return is around zero with only small deviations (say between £ 4 miln and £ 4 mln) but on the right side there is a small probability to have large losses (say a 2% probability that the loss is around £ 350 mln.). VaR will see no difference, VAR might notice it a little but only a coherent risk measure –such as ES– is bound to see the difference. So, a bank that is using only VaR will be* motivated *to take the large loss on board because the potential returns will be high –the market is not blind for the risk, onlyl the one using only VaR– and this high potential return will not be offset by higher capital demand because VaR will not see the risk.*

5.2 Risk Modeling

Now that we have shown that there exists at least one risk measure that is worth this title, the next problem is how to estimate this value (or any other parameter that we deem relevant). A large infrastructure project is typically one that is unique in many aspects. Even if it is a project that has been done before, it will be unique in that period in that geographical space. So we must be careful using the law of large numbers and convergence to a levy distribution under convolution –as Bouchaud & Potters (1997) warn us. First of all: there are no large numbers (even the largest bank will only have a limited number of projects), and hence there are not too much convolutions to be made.[18] Secondly it is very hard to find relevant(!) historic data. This is not only because there aren't so many similar projects, but most importantly all large projects will be unique. Even building twice the same nuclear power station in China will be a unique challenge because the few decades difference make in the second case the technology better known, but some skills harder to find, more or less administration and regulation, maybe there is a global economic crisis that separates the projects, etc.

So, in order to get an idea on the possible outcomes of the project one needs at least a zero hypothesis: this is called "the business plan". These are the official numbers put forward by the SPV that seeks the loan. It will be a detailed plan of cash-flows leading to construction and then incoming cash-flows from the exploitation of the project. Any investor can see (from his point of view of course) what each of the following parameters is:

- **time to profit**: how long does it take before the investment (SPV) makes profit
- **time to break even**: how long does it take before the investments equal the income
- **NPV**: the sum of all discounted cash flows $NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$
- **IRR**: internal rate of return or the discount rate that makes the IRR zero. In case the investments are just one up-front investment, then it equals the discount rate at which the present value of all future cash flow is equal to the initial investment or in other words the rate at which an investment "breaks even". Still otherwise stated, the IRR of an investment is the discount rate at which the net present value of negative

[18] The relevance lies herin that the convergence to a Levy distribution or Gaussian in most cases is slow, and actually never (unless in the mathematical case of infinity) work for the tails.

cash flows of the investment equals the net present value of the positive cash flows.

- etc.

This can be done in a simple spreadsheet such as LibreOffice Calc[19]. This simple approach already gives a good approximation of what is to be expected.

Now, assume that we consider IRR as the most relevant parameter and find a results of 14%. Is that a good or a bad result? In order to answer that question one has to compare the return with the risk. But in order to calculate the risk (for example ES) it is necessary to have a good idea how the returns are distributed. That is not so straightforward for large projects that are all quite unique.

The best method is of course starting from the underlying distributions calculate the distribution of the end-result. In practice, however this is not possible for project finance: distributions are unknown, non-linear interactions are possible, some events depends on others, etc.

The first, most simple, but most limited method is "stress testing".

5.2.1 Stress Testing

In order to gain some insight in how robust a certain result of our business plan is or what bad cases can be expected a simple stress test can answer that question.

A simple example could be: allow the price of certain raw materials to fluctuate (simply test a few possibilities), then do the same with labour prices, allow the effect of a strike, an earthquake, fluctuations in exchange rates, one of the lenders that gets into problems, we have to halt digging because we stumbled upon a site of historic importance, etc.

Soon, one of the problems with stress testing become obvious: it becomes bewildering how much possibilities there are, it is impossible to say which is more probable that the other, etc. The answer to that shortcoming is simply to restrict stress testing to what it does best: explore extreme risks –without knowing how likely it is. So for example assume that an earthquake destroys a lot of the half-build site, killed a few people causing a strike, the currency to plunge and the domestic bank in the syndicate gets into problems because of that. Then we have just one scenario, something that we can calculate with your spreadsheet and that gives us a "worst case scenario".

[19] LibreOffice is community-driven and developed software, and is a project of the not-for-profit organization, The Document Foundation. See: `http://www.libreoffice.org/`.

The relevance for each lender is that he should ask the question "can I afford to loose that much", if the answer is "no", then the lender should seek another partner in the syndicate, the sponsor another partner for the SHA, etc. Failing to do so is planning for the next Global Meltdown to start.

In order to do that in practice, a spreadsheet might stillbe sufficient, however, it might be advisable to follow a few simple rules to keep it organized. For example:

- Use different tabs (sheets) for (a) assumptions, (b) costs, (c) income, (d) P&L and (e) ratios.
- Make sure that each sheet has the same columns (the columns should be something like A, B, C and D hold titles, E is 2015-Jan, F is 2015-Feb, etc.
- Use different colours to make the different function of each cell clear: for example pale yellow for an input cell, no background for the result of a calculation, etc.
- Avoid –where possible– obscure formulae that are difficult to read for humans
- Do use as much as possible underlying programming language (Visual Basic for example) and never ever use marcos (macros are very difficult to read by other humans, not re-useable, slow and confusing).
- Keep different version, have frequent team-meetings when working on one file and agree who will modify what.

Following these simple rules will help you to make rather complex models in the simple spreadsheet that a modern computer offers. The downside of a stress test is that it does not tell how likely a certain result is. To get that essential insight it is necessary to use other techniques.

If you find that the spreadsheet becomes difficult to read or slow we suggest to have a look at the alternatives presented in Chapter 5.2.3 on page 167

5.2.2 Monte Carlo Simulations

A Monte-Carlo simulation can simply be understood as hundreds of thousands of Stress Tests run by an automated machine so that it becomes possible to get an idea about how probable certain outcomes are. This is of course only possible if we are able to say something sensible about the underlying risk factors.

With “something sensible” we mean that we know something about the likelihood of something to happen. We might not know the exact distribution, but at least some probability. For example we might expect an earthquake of force 4 to happen once in thousand years. This simple number is far less than knowing the probability density function, but it can already work.

In that case we would have a 0.000083 probability each month that such earthquake would occur. However if it occurs, then the knock-on effects will be significant for the project: damage, delays, other problems in the region needing attention, etc. It is here that the limitations of a spreadsheet become all too clear. It becomes impossible to model correctly the effect of such events, not only because of the interdependence with other parameters, but also in time. If such event occurred, then is it more or less likely to happen again? Some effects will be immediate (such as if the currency drops 20% with respect to the currency that we use to pay a certain material or service, then that service or material is immediately more expensive). This can still be modeled in a spreadsheet, but in the realistic case with the earthquake one must take into account a whole different scenario for the rest of the project and that becomes almost impossible and at least very convoluted.

The alternative is to use a programming language that allows us to model anything. Best suited for large projects are languages that allow for some object oriented code. We can use the features of an object oriented programming language to represent actors and input in our project. For example the engineering company can be one “object” and it will decide to hedge currency risk if the exchange rate hits a certain barrier, etc.

This allows us to model dependencies such as in our example with the earthquake. If the earthquake happened, then other objects can “see” that and react accordingly, the exchange rate (also an object) will switch regime (ie. draw its result from a different distribution), the workers can see the impact of the safety conditions and consider a strike with a given probability, etc. This way of working is not so far removed from the way modern computer games work.

Good examples of programming languages that allow vast amounts of complex calculations are C++ and R. The high level of abstraction offered by object oriented programming languages allows the programmer to create objects that can interact with each other and their environment. For example the Engineering Company can be such object. That object can be instructed to employ more workers when a delay threatens to happen but up to the limit that the extra costs are offset by the potential penalties. As the simulation then runs, market parameters change and events happen according to their probability of occurrence and each object will then interact in a pre-programmed or stochastic way.

This allows very complex behaviour and dependencies to be modeled, yet everything will be in a logical place and any other programmer can read it as a book. On top of that there are good free solutions to create free a professional documentation set. For example Doxygen (see `http://www.doxygen.org`) is free and able to create both an interactive website as well as a LaTeX[20] book for the documentation, that details each class, function, handle, property, etc. Code written in such way and documented properly is not only easy to maintain, but also straightforward to audit and as a bonus one gets the speed of C++.

5.2.3 Beyond the Monte Carlo Simulation

Now that we have a good idea how the distribution of the results will look like, we can use this distribution to calculate the relevant risk parameters. In many cases the "historic" distribution that we got by our Monte Carlo simulation will be usable, however for large and complex projects the distribution might not be very smooth. If we believe that this is a sign of the limited number of simulations, then we can try to apply a kernel estimation in order to obtain a smoother results that yield more robust risk parameters.

The technique of kernel density estimation (KDE) could be helpful for all distributions that are estimated from a histogram. As an alternative to parametric estimation where one infers a certain distribution it avoids the strong assumption that the data indeed follows that given distribution. Note a KDE can be used also for any input parameter where the distribution used is based on observations.

Of course one can choose a standard distribution if the we have reasons to assume that this would be a good approximation. However, choosing a non-parametric kernel density estimation, has the advantage of avoiding any assumptions about the distribution, and on top of that:

- it is well documented in the case of expected shortfall—(e.g. Scaillet 2004, Chen 2008, Scaillet 2005, Bertsimas, et al. 2004)

- there is research on its sensitivity with respect to the portfolio composition, $\boldsymbol{w}$—(see Scaillet 2004, Fermanian & Scaillet 2005)

[20]LaTeX is a high-quality typesetting system; it includes a large set of features designed for the production of technical and scientific documentation. LaTeXis the de-facto standard for the communication and publication of scientific documents. LaTeXis available as free software in the repositories of your distribution and at `http://www.latex-project.org/`.

Using a non-parametric kernel density estimation, however, requires one arbitrary parameter: "the bandwidth". The bandwidth is a parameter that is related to the level to which the data sample is representative of the real underlying distribution. If one makes a too-small choice of this parameter, one forces the estimated distribution function, f_{est}, to stick too much to the data, and there is too little of a smoothing effect. If, on the other hand, the parameter is insufficiently restrictive, then f_{est} will be smeared out over an area that is too large.[21] More information on bandwidth selection can be found in Jones, et al. (1996b).

Of course, one can ask if it is necessary at all use a kernel estimation instead of working with the histogram obtained from the data. Using the histogram as pdf has a few disadvantages:

- it is not smooth (this observation tells us that the use of histograms is similar to noticing that the dataset is imperfect and not doing anything about it),
- it depends on the end points of the bins that are used (changing the end points can dramatically change the shape of the histograms),
- it depends on the width of the bins (this parameter can also change the shape of the histogram),
- it introduces two arbitrary parameters: the start point of the first bin, and the width of the bins.

An answer to the first two points (and half of the last point) is to use a kernel density estimation. In that procedure, a certain function is centred around each data point (for example, an indicator function, a Gaussian distribution, the top of a cosine, etc.), these functions then are summed to form the estimator of the density function. The kernel density estimation is currently the most popular method for non-parametric density estimation (see e.g. the following books Scott 2015, Wand & Jones 1994, Simonoff 2012)

This method consists in estimating the real (but unknown) density function $f(x)$ with

$$f_{est}(x;h) = \frac{1}{N}\sum_{n=1}^{N} K_h(x-x_n) = \frac{1}{Nh}\sum_{n=1}^{N} K\left(\frac{x-x_n}{h}\right) \tag{5.20}$$

where K is the kernel

[21] Note that we do not use the usual notation for the estimated distribution density function, $\hat{f}$, because we have reserved that notation for the Fourier transform.

Definition 5.2.1 (Kernel). A kernel is a function $K(x) : \mathbb{R} \mapsto \mathbb{R}^+$ that satisfies the following conditions.

$$\begin{cases} \int\limits_{-\infty}^{+\infty} K(u)\,\mathrm{d}u = 1 \\ \forall u \in \mathbb{R} : K(u) = K(-u) \end{cases}$$

If K is a kernel, then also $K^*(u) := \frac{1}{h} K\left(\frac{u}{h}\right)$ (with $h > 0$) is a kernel. This introduces an elegant way to use h as a smoothing parameter, often called "the bandwidth".

This method was hinted by Rosenblatt et al. (1956) and further developed in its actual form by Parzen (1962). The method is thus also known as the "Parzen-Rozenblatt window method"

The Epachenikov kernel (see Epanechnikov 1969) is optimal in a minimum variance sense, however it has been shown by Wand & Jones (1994) that the loss of efficiency is minimal for the Gaussian, triangular, biweight, triweight, and uniform kernels.

We believe it if we believe that an underlying pdf exists, kernel density estimations have a few distinct advantages over histograms: they can offer a smooth density function for an appropriate kernel and bandwidth, and the endpoints of the bins are no longer an arbitrary parameter (and hence we have one arbitrary parameter less, but still the bandwith remains an arbitrary parameter).

We also note that Scott (1979) proves the statistical inferiority of histograms compared to a Gaussian kernel with the aid of Monte Carlo simulations. This inferiority of histograms is measured in the L^2 norm, usually referred to as the "mean integrated squared error" (henceforth MISE), which is defined as follows.

$$MISE(h) = E\left[\int\limits_{-\infty}^{+\infty} \{f_{est}(x;h) - f(x)\}^2 \,\mathrm{d}x\right] \tag{5.21}$$

A variant of this, the AMISE (asymptotic version), can also be defined, and this allows us to write an explicit form of the optimal bandwidth, h. Both measures have their relevance in testing a specific bandwidth selection method, for example. However, for our purpose these formulae cannot be

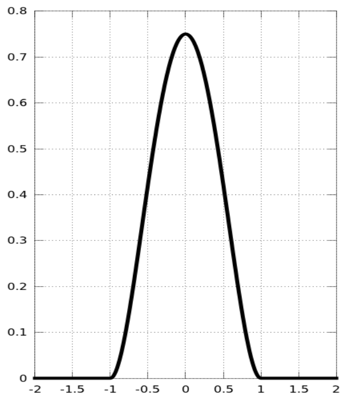

Figure 5.10: *Left, the Epachenikov kernel,* $K_h^E(x) = \frac{3}{4h}\left(1 - \left(\frac{u}{h}\right)^2\right)\mathbf{1}_{\{|u/h|\leq 1\}}$ *for* $h = 1$*; and right the Gaussian kernel,* $K_h^G(u) = \frac{1}{\sqrt{2\Pi h}}e^{-\frac{u^2}{h^2}}$ *for* $h = 0.5$*.*

used since they contain the unknown density function $f(x)$. Many alternatives have been proposed and many comparative studies have been carried out. A first heuristic was called "cross validation selectors" (see Rudemo 1982, Bowman 1984, Hall, et al. 1992). Sheather & Jones (1991) developed "plug-in selectors" and showed their theoretical and practical advantages over existing methods, as well as their reliable performance. A good overview is in Jones, et al. (1996a).

Conclusion Kernel Estimation is a widely accepted and used method that has many advantages. However, it introduces the arbitrary choice of bandwidth and of type of kernel. However, we note a novel method that automates this selection without the use of arbitrary normal reference rules (see Botev, et al. 2010). We also note that it is blind to specific aspects, such as the boundedness of the domain of values (e.g. prices cannot become negative). Therefore the method has to be used with care, and preferably on non-bounded data (e.g. log-returns).

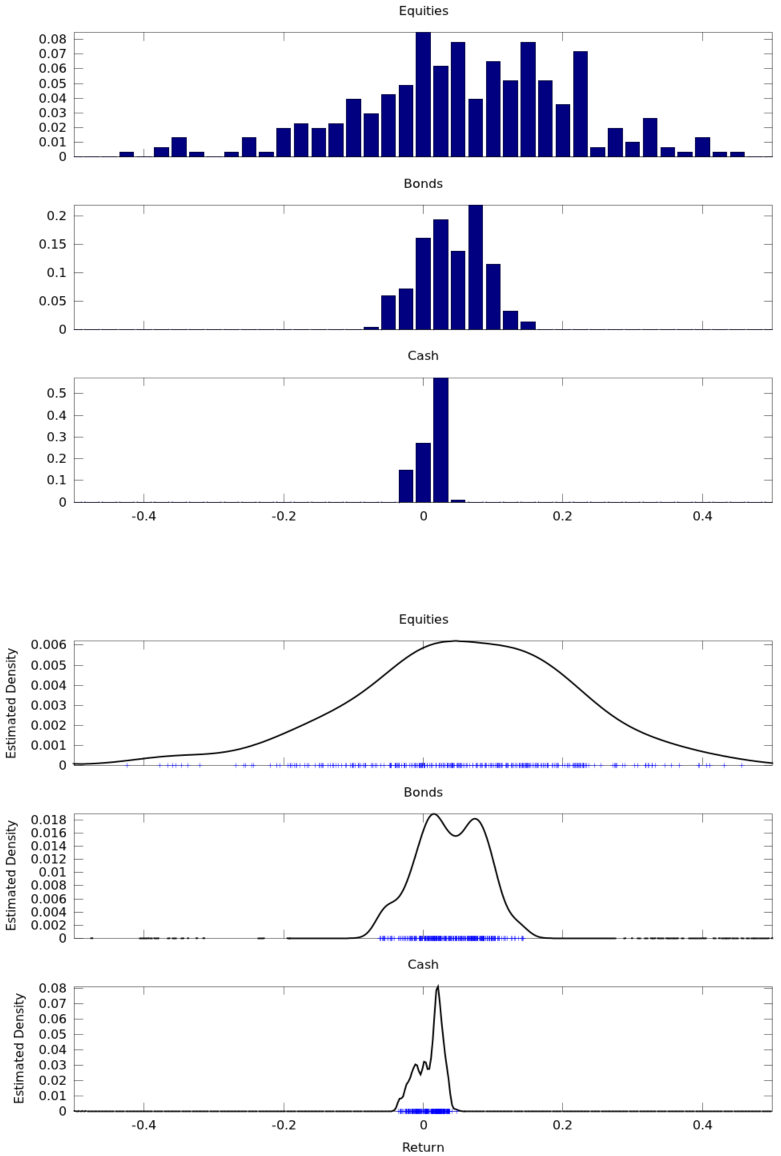

Figure 5.11: *As illustration on how the Epachenikov Kernels Estimation works, we present in the upper graph the histogram of the annual inflation corrected returns of standard asset classes. The lower graph offers a view on what a non-parametric kernel density estimation on those data can do.*

Chapter 6

Conclusion

In this part of the book we proposed that risk management is not only an essential part project financing, but that it is actually the reason why project finance exists. Therefore good risk management is key for all parties involved, each party has its own particular risks. What can be a risk mitigation for one party can be a new risk for another party. We proposed that risk management should follow a "risk cycle" and that the risk manager should continue to go through that cycle and continue to mitigate and improve the project.

Figure 2.1 on page 118 visualizes this idea –that the risk manager and his/her team should continuously cycle through the different aspects. For example the risk manager could decide to make weekly minor updates and monthly review all aspects involved (including the models, etc.). But even more essential than following a certain procedure is how practically oriented the risk manager is and how well he/she understands the big picture: the whole project. Once a risk is identified and assessed to be important, then he/she should go out and make sure that an appropriate mitigation is in place. In order to do so the risk manager will have to be involved in many decisions, be close to everyone and be a trusted partner for CEO and workers.

This section provided you with

1. a checklist of types of risks to look after in LRL (see Chapter 3 on page 121),

2. a simple way of visualizing, prioritizing and following up risks with a "risk matrix" (see Chapter 4 on page 133)

3. and all the mathematical tools to fill out this risk matrix (see Chapter 5 on page 137) — and if you would have skipped this part: the key message is that one should not ignore risk because they are very improbable.[1]

The reader will notice that there is no section on "risk mitigation". This is because risk mitigation is very specific in each situation and can only be assessed taking into account all specific human, engineering, political and environmental aspects. Therefore each risk mitigation will be highly specific to each project, company and even for each individual involved.

This section about risk management should give the risk manager a solid set of tools and a framework to do his/her job and communicate about it, but relevant practical experience remains quintessential. The most important aspect of risk management remains the way the risk manager (a) gains trust in the company in order to get all relevant information and (b) how he/she uses this information to mitigate risk before they realize. The risk manager should strive to be a trusted consultant to each person and team in the company and always keep the big picture in mind.

Though it is essential in the thinking about risk that there is nor free enterprise nor gain without risk. However there should be a healthy trade-off between risk and return: for a higher risk one should be able to expect a higher return. If a company were to plot its projects or loans in a risk-return graph, then it should look similar to the efficient frontier as presented in (consider the upper boundary of the plot in Figure 5.1 on page 141: an increasing curve). A company should within its risk appetite take only risks on board if the potential return is high enough.

While it is possible to make a some risks disappear (as eliminating explo-

[1]This remark refers to the fact that in the VaR is still used in regulations despite the fact that it totally ignores large-but-improbable risks. It seems remarkable that a risk measure that has proven to be very dangerous such as Value at Risk as it accompanied Lehman Brothers to its grave is still used. This seems to be similar to finding that it is hard to pick up mercury with a fork and then pass a regulation that forces bankers to use bigger forks. Arguably, a coherent risk measure –such as Expected Shortfall– might have been a small step in the right direction. However once more one would have to conclude that large infrastructure projects are always so unique that it becomes very hard to calculate any risk measure at all – and note that a coherent risk measure necessary needs more assumptions and/or knowledge about the tail of the distribution than VaR for example.

Indeed using a coherent risk measure in legislation is in our opinion only a "small step": we mean that a legislation such as Basel III creates inherently systemic risk and that this will not be solved by using a different risk measure. The problem is that a regulation that forces everyone to calculate in the same way and use the same risk limits is bound to create problems. Imagine that bank A hits its risk limit and has to sell investment X. That will put the price of investment X under pressure and that might in its turn push bank B over its limit, forcing it to sell X. This might lead to non-linear feedback system (in finance typically referred to as "systemic risk".

sion risk by closing a bottle filled with methane gas), no risk management is able to eliminate all risk of an infrastructure project. Recent years have seen increasing pressure on banks to become more safe organizations (Basel II and Basel III regulation for example — with Basel III especially tough on long term lending that is so much needed by infrastructure projects). Unfortunately these regulations make it difficult for banks to lend long-term and hence banks are less and less able to play their role in economic growth and project financing.

Probably the biggest risk that our society faces –especially in Europe– is the lack of economic growth induced by the de-leveraging of banks by and regulations such as Basel III. Perversely, the fear of the dilemma whether or not to bail out a bank has put the "taxpayer" directly liable for failing projects via Export Credit Agencies and the European Long-Term Investment Funds (ELTIFs).[2] Economic growth needs infrastructure projects and these are risky, that risk will never disappear: it can only be pushed to other parties or there will be less or no projects. If banks are not allowed to take on the risks, then other institutions will have to take that risk, otherwise the economy will stagnate.

It is unfortunate that humans only build traffic lights after accidents and that no-one ever has the courage to challenge existing lights. Regulators and law-makers only tighten regulation after a crisis . . . while the economy actually needs a much looser regime in order to recover. Maybe the availability bias –that causes short-term thinking– should be overcome and we should tighten regulation when the economy is doing great and –equally important– loosen regulation when economic growth is slow. This could even be built in into the regulation itself.

Will you have the courage and the long term view to start this?

[2]These –government owned– agencies tend to take over more and long term lending from banks. So when the project fails, the government is directly on the hook for the losses.

Part II

Addenda

Appendix A

Coherent Risk Measures

Note. This section is inspired on De Brouwer (2014).

A.1 Introduction

Financial institutions in general and banks in particular are –till now– the very heart of our free market economy. Banks help to put capital to use: they redistribute money from those who have cash but ran out of ideas or energy to those that have ideas and energy but insufficient cash. However, the Global Meltdown of 2008 has shown that they can be fragile. How was all that possible? How should one try to keep the risks in check? These questions are hardly discussed and law-makers simply assume that by doing more of what they did before it should all work out . . . till next time.

The essence of all commercial companies is to take risks and get a market premium in return. This also holds for banks and in a simple approach a bank can hold deposits (their liabilities) and give credits on the other side of their balance sheet (the bank's assets). Of course they will leverage this game in order to make more profit. So, the simple answer cannot be to make banks risk-less, because that would stall the economy and make them useless: risks are necessary but how can we control them?

Bernoulli (1738) recognized that the risk averse investor –and when banks invest in another legal or private person's debt, they are investors– should seek to diversify, Fisher (1906) proposed variance as a measure for economic risk, and Markovitz' PhD supervisor Marschak (1938) suggested to use mean and variance as parameters in utility (for consumption of commodities). However we had to wait till Markowitz (1952) for the first explicit quantification and identification of investment risk, when he proposed his mean-variance criterion. After almost 60 years his method is often used by practitioners and remains a reference for scholars (e.g Rubinstein 2002). Markowitz used in his seminal paper of 1952 variance as risk measure. He did not discus in that paper the choice for variance but argued that the mean-variance criterion was better than simply chasing return because it stimulated diversification: a wise and fundamental argument!

Now –70 years later and a few crisis wiser– the race is on to regulate Banks more and more strict in order to avoid a repetition of the meltdown in 2008. For example a series of "Basel" regulations try to de-leverage banks. The regulation tries to achieve this by using one risk framework and require banks stay within the limits set by that framework. This risk framework is based on Value at Risk (VaR) as a risk measure.[1] We will define it later, but first take a step back and look what scientific literature has available that one could have used instead.

A.2 Definitions

A.2.1 Coherent Risk Measures

Considering the question of how financial risk could be described, Artzner, et al. (1997) proposed the following set of axioms[2]:

Axiom A.2.1 (Coherent Risk Measure). A function $\rho : \mathbb{V} \mapsto \mathbb{R}$ (where $\mathbb{V}$ is the set of real-valued stochastic variables) is called a *coherent risk measure* if and only if it is

1. *monotonous*: $\forall X, Y \in \mathbb{V} : X \leq Y \Rightarrow \rho(X) \geq \rho(Y)$
   If a portfolio Y has always better results than portfolio X, then its risk is less.

[1] Enforcing one risk framework is risky, because it means that if things go wrong despite the risk framework (and statistically that is bound to happen) then many banks will be in the same dire streets for the same reason. This in itself works "systemic" and is dangerous. We do not focus on this discussion, but focus on the risk framework itself.

[2] For a profound elaboration and the proof that VaR is incoherent, please refer to Artzner, et al. (1999).

2. *sub-additive*: $\forall X, Y, X + Y \in \mathbb{V} : \rho(X + Y) \leq \rho(X) + \rho(Y)$
   Diversification reduces risk: the risk of a portfolio cannot be worse than that of any of its constituents separately.
3. *positively homogeneous*: $\forall a > 0$ and $\forall X, aX \in \mathbb{V} : \rho(aX) = a\rho(X)$
   Investing a times more money increases the risk a times.
4. *translational invariant*: $\forall c > 0$ and $\forall X \in \mathbb{V} : \rho(X + c) = \rho(X) - c$
   Adding an amount c in cash to the portfolio reduces the risk with c.

These axioms are congruent with what one intuitively understands as "risk" in a financial context. Axiom two for example states that diversification reduces risk. Indeed when one owns two assets, the outcome is in the worst case the result of the two worst case scenarios. In all other cases the outcome of the diversified portfolio is better. This is so fundamental in our thinking about financial risk that Artzner et al. elected this as an axiom for coherence and that Markowitz (1952) considered this aspect as the key test for his new mean variance criterion. Some authors, such as Acerbi & Tasche (2002a) consider only risk measures that are coherent in this sense and refuse to label other measures, such as Value at Risk (VaR henceforth), variance (VAR henceforth) and standard deviation, as "risk measure".

An important footnote is that these axioms are only realistic when markets are liquid and when the investor would never be in a position to influence price. This phenomenon –of limited validity– is not an argument against the axioms. This is so with most axioms such as for example the Euclidean axioms of geometry. This geometry generally work fine, but fails to describe reality in the presence of large masses—where one needs the theory of General Relativity (see Einstein 1916). Also, similar to the Euclidean axioms, this does not mean that this is the only set of possible axioms. For example it is possible to develop internally coherent geometries with different axioms, such as Lobachevsky and Bolyai's geometry on the surface of a pseudo-sphere, or the geometry on a sphere, developed by Riemann. While none of these sets of axioms can be wrong or right, only the Euclidean geometry makes sense for applications such as building a house or a printing a book. In the same way we will illustrate that Artzner et al.'s set of axioms lead to a coherent representation of risk in liquid financial markets for investors who cannot influence price.

A.2.2 Variance (VAR)

Variance is defined as follows.

Definition A.2.1 (Variance (VAR)). Let X be a real-valued stochastic variable, its variance is then defined as:

$$\begin{aligned} VAR &:= E[(X - E[X])^2] && \text{(A.1)} \\ &= E[X^2] - (E[X])^2 && \text{(A.2)} \\ &= \int_{\mathbb{R}} (x - \mu)^2 \, f_X(x) \, \mathrm{d}x && \text{(A.3)} \end{aligned}$$

with $\mu := E[X] = \int_{\mathbb{R}} x \, f_X(x) \, \mathrm{d}x$

Note that since standard deviation is a monotonous function of VAR that all the results in this paper that are obtained for VAR are also valid for standard deviation.

A.2.3 Value at Risk (VaR)

First we define Value at Risk (VaR).

Definition A.2.2 (Value at Risk (VaR)). For the real-valued stochastic profit variable, return (X), and a probability $\alpha \in [0, 1]$, the Value at Risk (VaR) is defined as

$$VaR_\alpha(X) := -q_\alpha(X)$$

with $q_\alpha(X)$ the α-quantile of the stochastic variable X.

Note that in the rest of the paper we will assume a "profit variable" when discussing risks (in line with practice in banking and asset management as opposed to practice in insurance where one typically considers the distribution of the "loss variable").

Often this definition is worded as "the VaR, given a confidence interval of $(1-\alpha)$, is that value so that there is a probability of α to have an outcome worse than VaR." In other words, the "α-VaR is the best outcome of the $100\alpha\%$ worst outcomes".

A.2.4 Expected Shortfall (ES)

Quite probably VaR is so popular for its simplicity, ease of calculation, universality, and seemingly (but deceivingly) ease of interpretation. VaR is the answer to the simple question: "What is the lowest amount that one can expect to loose in the $100\alpha\%$ worst cases?". Another advantage is that it is expressed in the same units as the portfolio, and hence can have some intuitive interpretation. In the years that followed Artzner et al. (1997)'s publication there was no coherent alternative for VaR known that combined all of the advantages that made it so popular.

Pflug (2000) proved that the answer to the following simple and question: "What is the average amount of loss in the $100\alpha\%$ worst cases?" leads to a coherent risk measure. This risk measure goes by many names: Conditional Value at Risk (CVaR), Expected Shortfall (ES), Average Value at Risk (AVaR), Expected Tail Risk (ETR). For continuous distributions it coincides with the Tail Conditional Expectation (TCE). For non-continuous distributions TCE is, however, not the same as ES. For non-continuous distributions it can even be non sub-additive.

We will choose for the name Expected Shortfall (ES henceforth), as it seems the most clarifying and pure and define ES as follows.

Definition A.2.3 (Expected Shortfall (ES)). For a real-valued stochastic profit variable, X (such as return), the Expected Shortfall is the expected loss in the $100\alpha\%$ worst observations

$$
\begin{aligned}
ES_{(\alpha)N}(X) &:= -\frac{\sum_{i=1}^{n} X_{i:N}}{n} \\
&= \text{- (average of the worst } 100\alpha\% \text{ realizations)}
\end{aligned}
$$

Where the index i:N refers to the index of the observations, after sorting from the worst to the best observations, and n is a number to be determined in function of α.

(Acerbi & Tasche 2002b) show that this is equivalent with;

Definition A.2.4 (Expected Shortfall).

$$ES_{(\alpha)}(X) := -\frac{1}{\alpha}\int_0^\alpha F^{\leftarrow}(p)\,\mathrm{d}p$$

with $F^{\leftarrow}(p) := inf\{x|F(x) \geq p\}$, the left-generalized inverse of the cumulative distribution function of X. For continuous distributions one has $F_X^{\leftarrow}(p) = q_\alpha(X)$.

This definition makes immediately clear that ES is continuous in α. This is a property that has conceptual importance, and is not possessed by VaR, for example. Figure A.1 shows how counter-intuitive this makes VaR.

Corollary A.2.1. *For continuous distributions, this definition can be written as*

$$ES_{(\alpha)}(X) = -\frac{1}{\alpha}\int_0^\alpha Q_X(p)\,\mathrm{d}p \tag{A.4}$$

with $Q(p)$ the quantile function (or the inverse of the cumulative distribution function)

Which is then also equivalent to

$$ES_{(\alpha)}(X) = -\frac{1}{\alpha}\int_0^{q_\alpha(X)} x f_X(x)\,\mathrm{d}x$$

So, since 2000 we have a coherent alternative for VaR that has most of its advantages: simple interpretation, expressed in monetary unit of measurement and universality of use. However there is a drawback: ES is more difficult to estimate. Indeed, where VaR simply ignores tail of the distribution, ES will take all worst outcomes into account. This makes ES more sensitive to estimation errors in the tails of the distributions. However we note that there are many ways to simplify the calculations.[3] Also note that for a given probability, ES is always higher or equal than VaR.

[3]For portfolio optimization and risk minimization we refer to (see for the fundamentals: Rockafellar & Uryasev 2000, Rockafellar & Uryasev 2002, Uryasev 2000, **?**) and (for overviews De Brouwer 2012, Acerbi & Simonetti 2002, Strepparava 2009); for comonotonic approximations: (**?**, see)]vanduffel2005comonotonicity,vanduffel2008optimal,dhaene2007comonotonicity,dhaene2005comonotonic,van

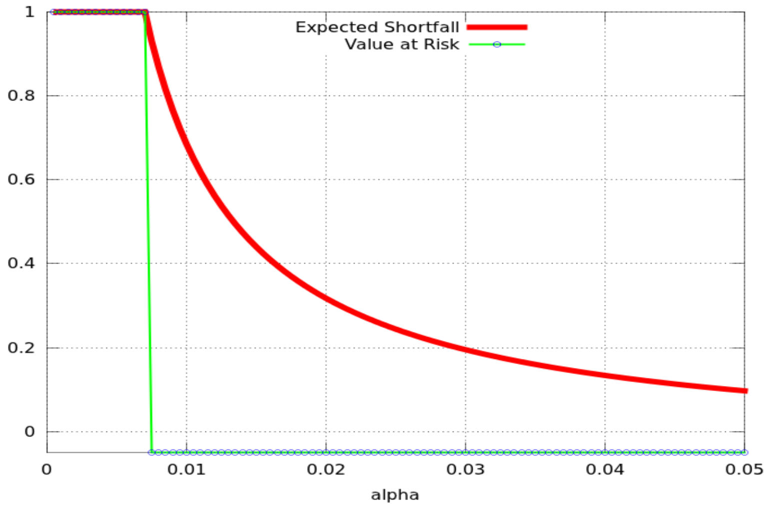

Figure A.1: *The $VaR_{0.01}$ and $ES_{0.01}$ in function of α for one bond as defined in Example A.3.1. If one believes that VaR is reasonable, then it is reasonable to see the risk drop from 100% to -5% when the desired confidence level passes from $(1 - 0.0069999)$ to $(1 - 0.0070000)$. It is reassuring that ES is continuous in α.*

More importantly, one can consider the question if it is better to have a less stable calculation or is it better to neglect the highest risks all together as in a VaR calculation. We believe that the answer to that question is obvious and in favour of ES.

A.2.5 Spectral Risk Measures

A simple and elegant proof for sub-additivity of ES for discrete distributions is presented by (De Brouwer 2012) (pages 124 and 125). For continuous distributions the proof is generally given via Distortion Risk Measures or via Spectral Risk Measures. Both representations are equivalent.

In general, we can define spectral risk measures as follows.

Definition A.2.5 (Spectral Risk Measure). Let X be a stochastic variable, representing the return of a financial asset. Then we define the **spectral measure of risk** $M_\phi(X)$ with **spectrum (or risk aversion function)** $\phi(p) : [0,1] \mapsto \mathbb{R}$ as:

$$M_\phi(X) := -\int_0^1 \phi(p) F_X^{\leftarrow}(p)\, \mathrm{d}p$$

Example A.2.1. The spectrum or risk aversion function for the α-Expected Shortfall (ES$_\alpha$)—as defined in next section—is

$$\phi_{ES_\alpha}(p) = \frac{1}{\alpha}\mathbf{1}_{[p\leq\alpha]} := \begin{cases} \frac{1}{\alpha} & \text{if } p \leq \alpha \\ 0 & \text{else} \end{cases} \tag{A.5}$$

Example A.2.2. The spectrum or risk aversion function for the α-VaR is the Dirac delta function:

$$\phi_{VaR_\alpha}(p) = \delta(p-\alpha) \tag{A.6}$$

The spectral representation of risk measures clarifies a lot.[4] Via this presentation it is possible to determine necessary and sufficient conditions on the spectrum for a risk measure to be coherent.

[4]More details can for example be found in De Brouwer (2012), (Acerbi 2004) and most of the proofs are in Acerbi (2002).

Theorem A.2.2. *The risk measure $M_\phi(X)$ as defined above is coherent, if and only if*

$$\begin{cases} \phi(p) \text{ is positive} \\ \phi(p) \text{ is not increasing} \\ \int_0^1 \phi(p)\,\mathrm{d}p = 1 \end{cases}$$

Proof. *(see Acerbi 2002)*

□

This theorem proves that there is a deep relation between what we have defined as "coherent" and a non-increasing risk spectrum. A not-increasing risk spectrum means that in calculating the risk measure, one cannot assign a lower weight to a worse outcome. In other words the spectrum $\phi(p)$ of the risk measure M_ϕ determines the weights associated to possible outcomes. This explains the alternative name for $\phi(p)$: "risk aversion function".

A person who thinks coherently cannot allocate a higher weight to better outcomes in a risk measure, and hence the risk aversions function is not increasing. And this is what VaR does: it assigns a zero weight to all outcomes worse than a certain quantile: in Equation A.6 one can see that the spectrum of VaR increases infinitely steep just before α.

A.3 The Consequences of Thinking Incoherently

Let's illustrate the consequences of failing to select a coherent risk measure with a simple example. Let us start with an example, similar to the one used in Acerbi (2004).[5]

Example A.3.1. Assume that we're running a small bank and got already deposits and we need to decide on something for our the asset side of our balance. Our preference goes to a bond that with a probability of 99.3% pays in one year a coupon of 5% plus the principal. So, there is a 0.7% probability to default –in which case no payout is expected.
Its 1% VaR is -5%, because the only default cases are below the 1% quantile. In other words, VaR spots *no* risk!

[5] For the reader it might be comforting to note that the results that we present here are exactly the same as those presented by Acerbi, even while our method of calculating was very different. We used a numerical approach by convolving the underlying distributions and Acerbi (2004) used binomial distributions.

Example A.3.2. Now consider two identical bonds with the same parameters, but independently distributed (say, one bond on a French real estate company and another on a Canadian diamond mine).
Clearly a portfolio that holds 50% of each must be less risky. Only in the extreme improbable case (probability = 0.000049) that both default we lose all our money, in all other cases at least 50% of the portfolio is preserved. Diversification reduces the risk.
However calculating the 1% VaR of the diversified portfolio one finds that it equals 47.5% of the portfolio! This is a lot higher than in Example A.3.1!

Example A.3.3. Consider now N such identically and independently distributed bonds.
VaR and ES behave then—in function of N—as presented in Figure A.2.

Without loss of generality but for clarity's sake we assumed that all assets (bonds) have similar characteristics. We –as bankers– have now to decide what to do: buy one bond, 2 or more. Since Basel II our risk manger is forced to calculate risks based on VaR with on a one year horizon with a confidence level of 1%. His conclusion is bound to be: "gentlemen, we should only invest in one bond: let's go for the French real estate". And the rest is history: how different is our story from what happened to Lehman Brothers?[6]

This is a dramatic example of what can happen when one of the axioms of coherence is not respected. VaR is not sub-additive, and hence can and will lead to absurd results. Using VaR to measure risk is similar to using a rubber band in order to measure length. Because of the lack of sub-additivity VaR can counteract diversification!

It has to be said, however, that when distributions are "nicely bell-shaped"[7], VaR will give a good indication of risk.

In Figure A.2 we present an overview of what happens to the VaR when the portfolio is more and more diversified. There is not only a false minimum at one bond, but also at 21 and 63! These false minima are not only counter-intuitive, but make VaR useless for portfolio optimization and risk

[6]Some insight in the balance sheet of Lehman Brothers can be found in McDonald & Robinson (2009), or in its annual statements of accounts.

[7]A mathematical correct way of expressing this "nice bell-shape" is via elliptical distribution functions: VaR is sub-additive for elliptically distributed returns. An elliptical distribution function is defined as a distribution for which the characteristic function only depends on one variable: $\boldsymbol{x}'\boldsymbol{\Sigma}\boldsymbol{x}$, where $\boldsymbol{\Sigma}$ is a positive definite matrix. A characteristic of these distribution functions is that in the two-dimensional case the iso-density curves form ellipses (and in higher dimensions ellipsoids), however this property is not a sufficient condition to be an elliptical function. The multivariate normal, logistic, exponential power, Cauchy, Laplace and t-distributions for example belong to the class of elliptical distributions.

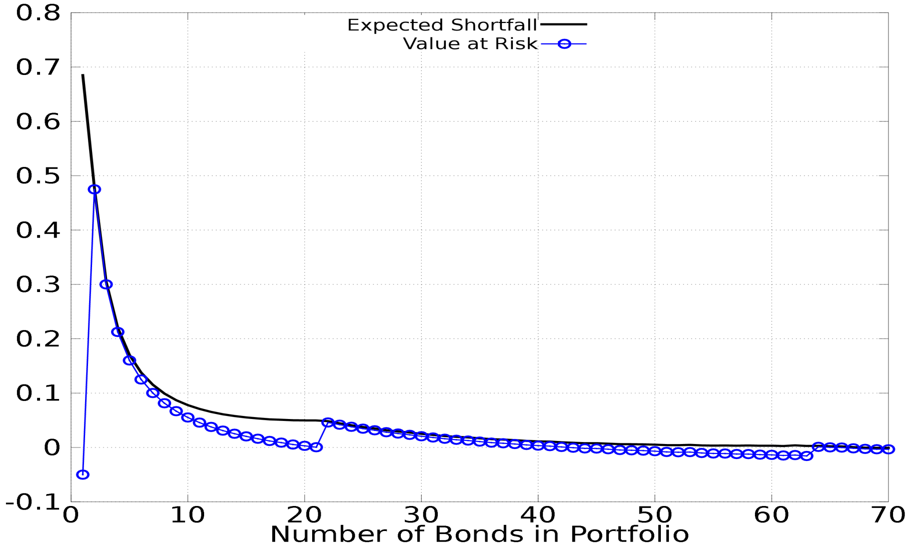

Figure A.2: *The risk associated to Example A.3.3. On the x-axis we present an increasing number of independent bonds in the portfolio. On the y-axis the fat line without dots is* $ES_{0.01}$*, the one percent expected shortfall—see Definition A.2.3, and the narrow line with dots is the* $VaR_{0.01}$*. Note that VaR displays a false minimum for 1, 21 and 63 bonds and that ES is always higher or equal to VaR.*

minimization. When the risk surface is not concave, then it is not possible to make an optimization and trust the results. This in itself means that VaR is a flawed concept that has only in some cases accidentally something to do with risk. Mathematicians will express this as “a risk measure has to be convex”.

One might argue that the point of view is important. Indeed, let us rethink the previous examples, and assume that we –as banker– are not choosing bonds for our own portfolio, but rather for the portfolio of our clients (ie. we’re advising). Assume further that the client will withdraw all his assets from us if a default occurs. In this case the banker could decide to minimize the probability that a loss occurs and just pick one bond. This is of course very irresponsible, but worst of all we can now argue that we minimized VaR for our client!

The axioms under which VaR would be coherent still have to be written. It seems to us that monotonicity, positively homogeneity together with a third axiom that states that all outcomes below a certain probability level are lethal (so that the amplitude of loss is irrelevant) could be a possible set of axioms under which VaR is coherent. A person with a sound mind would however not find that these axioms describe his or her thinking about financial risk in a sufficient accurate way. Furthermore this does not solve the problem of lack of convexity.

A.4 Portfolio Optimization and Risk Minimization

Banks have to build a portfolio of assets and since Basel II forces them to use VaR as risk measure, they might be inclined to use it also as a risk measure to optimize the portfolio. The lack of sub-additivity (and hence lack of convexity) of VaR however makes this a hazardous enterprice.

A.4.1 Value at Risk (VaR)

Example A.4.1. Consider the risk surface (efficient frontier) of Example A.3.3.

A numeric optimization using VaR will converge to a local minimum, such as one, 21 or 63 bonds. When a coherent risk measure, such as ES, is used the “most safe” portfolio is the one with best attainable diversification (so for an infinite number of independent bonds). We refer to Figures A.2 and A.3 for two different views on this problem.

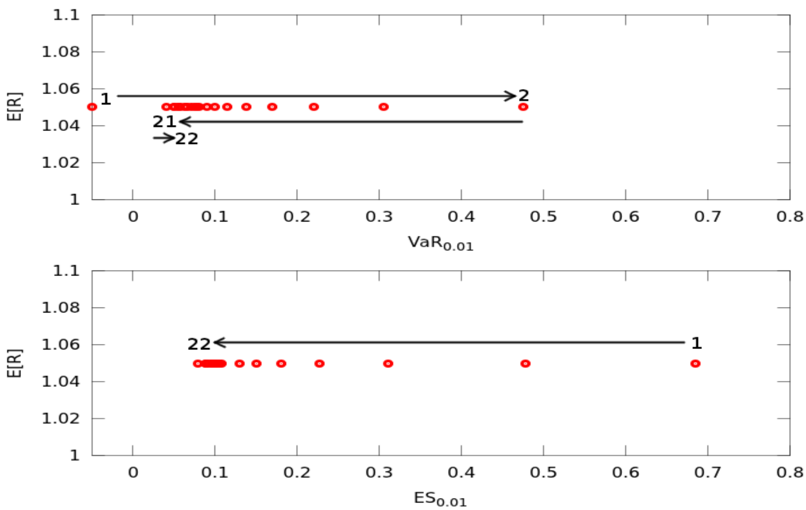

Figure A.3: *The risk surface of the $VaR_{0.01}$ and $ES_{0.01}$ of Example A.4.1. The numbers—next to the dots—correspond to the number of independent bonds in portfolio. ES will decrease as diversification increases, VaR behaves unpredictable in function of diversification.*

Also here we see that a coherent risk measure finds a logical answer, an incoherent risk measure can and will give rise to surprises.

A.4.2 Variance (VAR)

One might be tempted to use the good old variance (VAR henceforth) in order to optimize portfolios because it is sub-additive and hence convex. However it fails another important test: it is not monotonous.

Let us consider the following example:

Example A.4.2. Consider an asset A with expected return 5% and a standard deviation of 2%, also the return is never lower than 1%. Next consider an asset B with an expected return of 0% with a standard deviation of 0.5% and with a return that never exceeds 1%. In this example the return of A will always be higher than that of B but the volatility of B is lower.

The two assets in Example A.4.2 are very different A could for example refer to a government bond and B to a derived construction on that bond (that includes way too much (hidden) costs but reduces volatility). VAR will consider investment B as the least risky and comparing risk and return (in the sense that Markowitz (1952) proposed) we must conclude that both are valid investments: the one with the lower return (B) has also the lower risk. However let's look more carefully: asset B is is not less risky but it is clearly more risky: it is an investment where one has a a significant chance to loose money (depending on the exact shape of the distribution this probability be expected to gravitate around 50%) where A offers a sure profit that is always higher! For all reasonable people (that prefer more money over less and consider losing money as a risk) B is more risky than A, but VAR tells us the opposite!

We acknowledge that in reality it would be rather seldom to see investments A and B together in the market, but we argue that this is no reason do discard this phenomenon. A risk measure should be universal and coherent and not only applicable to assets that one sees today. Furthermore while this example is a little extreme, it is trivial to construct a less crude and clear example that illustrates the same issue.

A.5 Regulatory Use of Risk Measures

A.5.1 Using VaR as a Risk Limit

Since version II of the Basel regulations Value at Risk (VaR) is the prescribed risk measure. While this has some advantages over volatility (VaR is meaningful for the larger class of elliptical distributions and not only relevant for the Gaussian distribution and VAR is monotonous) we have shown before that it has at least one critical drawback which makes it very fragile as risk measurement: VaR is not sub-additive (and not convex).

Of course we support the thesis that a regulation has to cover all banks and not assume a well diversified portfolio of assets, but may we hope that the lack of sub-additivity will be irrelevant for the larger and well diversified portfolio of assets of that is so typical on the balance of a bank? Many risk managers are so used to work within the regulatory framework and got so accustomed to large portfolios of assets (eg. a portfolio of 2 million retail loans) that they will rather not expect any problems with sub-additivity.

However the case of Lehman Brothers proves that VaR calculations are not a guarantee against bankruptcy. Indeed the argumentation that a well diversified portfolio should have no problems with sub-additivity is misguided. The Central Limit Theorem (see for example Bouchaud & Potters (1997)) only works for the centre of the distribution. Heavy tails can indeed cause VaR to be incoherent.

A.5.2 What about Stress Tests?

It might be argued that stress tests are the least scientific and most naive method of assessing risk. While there is some truth in this, it must be noted that a stress test gives us insight in "what can happen" (without any concept of probability linked to that scenario). And there lies the value of stress tests. When one uses VaR, one bluntly ignores the extreme risks and stress tests can show us what can happen in such extreme cases.

A mathematician will argue that a stress test has a "weight of zero", in other words that it has a probability of zero to realize itself and that we cannot even assess how likely it is to end up in the neighbourhood of this extreme scenario. While this is true, stress tests are a valuable addition to any risk measure. Any risk measure will be a one dimensional scalar by definition and unlikely to grasp the whole complexity of the real world. Even a coherent risk measure is only one possible reduction of reality.

A.5.3 How could banks be safer?

A fundamental, powerful and essential aspect of our capitalistic economy is the concept of allowing many people to share risks and doing so transferring money from those who have money to those who have ideas and want to work. Every milestone that deepened this concept (the bond, the traditional bank, joint stock company and the stock exchange) was a turning point in history and allowed kings and nations to gain an essential competitive advantage for a while and created a lasting leap forward in the world economy.

Assuming[8] that p2p (peer to peer) lending and crowd-sourcing will not replace banks anywhere soon in providing this service we must conclude that banks are very important for the economy and scientist, law-makers and regulators must do all they can to keep this essential service operational. In this section we will focus on banks that are mainly providing this service to our economy and will refer to them as "traditional banks". The so called "investment banks" that get a large portion of their income from trading are actually operating as hedge funds and will not be discussed in this section.

The first thing that should be done is making a clear distinction between traditional banks and the ones that operate as hedge funds. Institutions that operate as traditional banks keep the motor of our economy running and should be subject to a specific regulation. This regulation should measure risks in a coherent way. Doing so we would create an environment that is much safer than the VaR approach that stimulates taking large-but-improbable risks on board.

The second issue to tackle would probably be the fact if one can enforce one risk measure for all banks, because that in itself can work systemic. This important question is outside the scope of this paper.

A.6 Conclusions

We already mentioned that the title of this paper is a reference the seminal paper of Artzner et al. (1997): 'Thinking Coherently", however with hindsight it might also be a reference to Oscar Wilde's play "The importance of being Earnest". Since (Artzner et al. 1999) we know that VaR is not coherent, but we choose to believe the fictional character of VaR, ignore its true incoherent nature and both practitioners and regulators embrace more and more VaR.

[8]P2p lending is in the last four years making its comeback after banks dominated the lending landscape roughly since the creation of the Medici bank. However, in the few countries where it has some success –such as USA and UK– it's market share in retail lending is almost negligible.

The result is that Basel II created the mindset that is unequivocally connected to VaR: underestimate or bluntly ignore the extreme-but-improbable risks. Doing so it paved the way to the Global Meltdown of 2008. Fortunately law-makers and regulators sense that something is wrong and in order to compensate the weaknesses of VaR they add loads of other tests and regulations. This over-regulation of course creates costs that are paid by customers and slows down the economic recovery. However, more importantly it is dangerous as it makes it really difficult to see what is really going on and because no matter how vast the over-regulation will be it will never be as strong as one simple coherent risk measure or principle.

VaR is not an acceptable risk measure because it disregards the tail of the distribution completely (and as a related matter it is not sub-additive nor convex). Financial crises and the Global Meltdown show us that the devil dwells in the left tail of the profit distribution. The problems with ES (more computationally intensive and less stable results) seem—to us—no excuse to completely disregard the tail of the distribution. Only when using a coherent risk measure, one will obtain coherent results.

Note also that the popular standard deviation is only a measure for dispersion, not a measure for risk. This is because standard deviation and variance attribute an increasing contribution to better outcomes, which is not logical for a risk measure to do.

By constructing examples, we have proven that both VaR and VAR are incoherent risk measures—not only in a mathematical sense but also in a semantic sense—and hence can and will lead to illogical results. By extension, any method based upon them is inconsistent and flawed. Markowitz (1991) remarked that "Semi-variance seems more plausible than variance as a measure of risk, since it is concerned only with adverse deviations" (p. 286); and since Artzner et al. (1997) there is proof that his insight is a deep truth. The examples presented in this paper clearly show that coherence matters not only in the minds of mathematicians, but also in the portfolio selection in real life.

As an alternative we suggest to consider ES, because it is the most simple coherent risk measure of the class of distortion risk measures (see De Brouwer (2012), p. 124).

Important Notices

1. It is not the author's intention to assess the work done for the Basel regulations as a whole. This is an entirely different discussion that is much wider than the risk measure alone. In this paper we only scrutinize

the choice of risk measure.

2. Note that also a coherent risk measure is also just one view via one frame[9] on the complex reality of the modern financial markets. Even a view through a "coherent window" necessarily reduces the multidimensional, complex and interlinked reality to $\mathbb{R}$. This is always a reduction of the true richness of the dangers on financial markets, necessarily based on information from the past and it will never replace risk management by humans, nor will it prevent (previously unseen) adverse moves in financial markets!

3. Recent work around "elicitable risk measures" (see (Bellini & Bignozzi 2013) and (Ziegel 2013)) emphasizes the importance of having an easy way to "back-test" a risk measure (ie. verify if the number used is is –still– appropriate) on a limited set of data. This is what financial institutions try to do now with VaR calculations. The argument is that "expert estimates" have to be limited as much as possible and that past observations should form an objective base of calculating capital reserves for example. However, we would argue that real risk of interest is the tail risk, and this is in essence unknown. We argue that a bank should be safe in case of *future* disasters (as opposed to be "99% safe" calculated on the last years history of data). This means that if one would have some tail observations relevant for calculating ES, one still would have to complement it with an estimate of some "Black Swan events". This manual correction is probably bigger than the instability of estimation of the tail-risk. This –we believe– largely overthrows the elicitability argument — though important in its own right! For example, Lehman Brothers had no chance to see this liquidity trap coming even when they would use ES based on historic data only! What they could have done is add something like "there is however a small probability of 0.05% that we loose 90% on this project" (it is stated as a delta-dirac function but that does not matter in our calculations the integral will "average it out"). The resulting ES will be a convolution of observed history and our "expert guess on extreme events". And last but not least: we prefer a risk estimate to be less stable and more difficult to back-test and even more sensitive to my assumptions on the tail of the risk than totally ignore the tail risk!

[9] "Frame" should be understood as in the sense described by Tversky & Kahneman (1981). Its concept would be close to a mindset, a subset of a larger problem and a point of view.

Appendix B

Levels of Measurement

Note: This section is based on De Brouwer (2012).

It is customary to refer to the theory of scales as having been developed by Stevens (1946). In that paper he argues that all measurement is done by assuming a certain scale type. He distinguished four different types of scale: nominal, ordinal, interval, and ratio scales.

B.1 Nominal Scale

The nominal scale is the simplest form of classification. It simply contains labels that do not even assume an order. Examples include asset classes, first names, countries, days of the month, weekdays, etc. It is not possible to use statistics such as average or median, and the only thing that can be measured is which label occurs the most (modus of mode).

Note that it is possible to use numbers as labels, but that this is very misleading. When using an nominal scale, none of the traditional metrics (such as averages) can be used.

B.2 Ordinal Scale

This scale type assumes a certain order. An example is a set of labels such as very safe, moderate, risky, very risky. Bond rating such as AAA, BB+, etc. also are ordinal scales: they indicate a certain order, but there is no way

Scale Type	Nominal
Characterization	labels (e.g. asset classes, stock exchanges)
Permissible Statistics	mode (not median or average), chi-square
Permissible Scale Transformation	equality
Structure	unordered set

Table B.1: *Characterization of the Nominal Scale of Measurement.*

to determine if the distance between, say, AAA and AA- is similar to the distance between BBB and BB-. It may make sense to talk about a median, but it does not make any sense to calculate an average (as is sometimes done in the industry and even in regulations)

Scale Type	Ordinal Scale
Characterization	ranked labels (e.g. ratings for bonds from rating agencies)
Permissible Statistics	median, percentile
Permissible Scale Transformation	order
Structure	(strictly) ordered set

Table B.2: *Characterization of the Ordinal Scale of Measurement.*

Ordinal labels can be replaced by others if the strict order is conserved (by a strict increasing or decreasing function). For example AAA, AA-, and BBB+ can be replaced by 1, 2 and, 3 or even by -501, -500, and 500,000. The information content is the same, the average will have no meaningful interpretation.

B.3 Interval Scale

This scale can be used for many quantifiable variables: temperature (in degrees Celsius). In this case, the difference between 1 and 2 degrees is the same as the difference between 100 and 101 degrees, and the average has a meaningful interpretation. Note that the zero point has only an arbitrary meaning, just like using a number for an ordinal scale: it can be used as a name, but it is only a name.

Scale Type	Interval Scale
Characterization	difference between labels is meaningful (e.g. the Celsius scale for temperature)
Permissible Statistics	mean, standard deviation, correlation, regression, analysis of variance
Permissible Scale Transformation	affine
Structure	affine line

Table B.3: *Characterization of the Interval Scale of Measurement.*

Rescaling is possible and remains meaningful. For example, a conversion from Celsius to Fahrenheit is possible via the following formula, $T_f = \frac{9}{5}T_c + 32$, with T_c the temperature in Celsius and T_f the temperature in Fahrenheit.

An affine transformation is a linear transformation of the form $\boldsymbol{y} = \boldsymbol{A}.\boldsymbol{x} + \boldsymbol{b}$. In Euclidean space an affine transformation will preserve collinearity (so that lines that lie on a line remain on a line) and ratios of distances along a line (for distinct collinear points p_1, p_2, p_3, the ratio $||p_2 - p_1||/||p_3 - p_2||$ is preserved).

In general, an affine transformation is composed of linear transformations (rotation, scaling and/or shear) and a translation (or "shift"). An affine transformation is an internal operation and several linear transformations can be combined into one transformation.

B.4 Ratio Scale

Using the Kelvin scale for temperature allows us to use a ratio scale: here not only the distances between the degrees but also the zero point is meaningful. Among the many examples are profit, loss, value, price, etc. Also a coherent risk measure is a ratio scale, because of the property translational invariance implies the existence of a true zero point.

Scale Type	**Ratio Scale**
Characterization	a true zero point exists (e.g. VAR, VaR, ES)
Permissible Statistics	geometric mean, harmonic mean, coefficient of variation, logarithms, etc.
Permissible Scale Transformation	multiplication
Structure	field

Table B.4: *Characterization of the Ratio Scale of Measurement.*

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