Thinking Coherently for Everyone

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Abstract

Niels Bohr once said that if all scientific knowledge would be lost and we were to pass only one sentence to the next generation that we should pass the information that matter is composed of atoms. With a hundred or so years more hindsight I would prefer another sentence to be passed on. I would propose "When thinking about risk and especially systemic risk in financial markets, do it in a mathematically coherent way, that is only use convex risk measures that are related to the downside of the profit distribution". I believe that our ability to start thinking coherently about risk will determine whether or not the capitalist system will survive and whether or not we leave a free world with some wealth in it to our children or not. This paper aims to illustrate why I consider this as so important.

1 Introduction

During last century thinking about risk has evolved dramatically: utility has been linked to risk and various risk measure have been proposed such as volatility. Back in 1997, Artzner, Delbaen, Eber and Heath, published a paper "Thinking Coherently". With financial risks in mind, but with a deep and universal approach, they prosed four axioms that risk measures should satisfy in order to be "coherent". To be honest, there might be more coherent sets of axioms, just as there are many coherent alternatives for Euclid's axioms for geometry. Just as these alternatives describe another reality than the geometry on the plane (for example geometry on a sphere), it is also in finance possible to find other sets of axioms that are coherent. What is thinking coherently in finance?

Different people might define coherence different and viewpoints and circumstances might differ, but few people would disagree that a coherent risk measure should satisfy the following conditions:

1. is an investment X has always worse outcomes than Y (in every observation), then X is more risky—in other words, a risk measure should be related to the downside of the returns.

2. If we invest more, our risk increases—Artzner asks a direct connection: twice the investment, then the risk is double.

3. If we diversify, the risk must decrease or at least stay the same—in other words risk cannot increase when diversification increases.

4. If we hedge our position with cash, then the risk decreases with that amount—Artzner asks that if we add an amount of cash, that the risk should be decrease with the same amount.

The above statements seem to be acceptable, but are not the only possible ways to define coherence. This definition of coherence seems to be fine for the investor that cannot influence prices, is not allowed to think in terms such as "once I default it does not matter any more". The investor in Artzner's approach cannot influence price and while an individual would not care if he has twice or ten times an amount that he or she can never repay, for the society it does matter. For example, once a bank defaults its shareholders lost their investment, and that's it. They might not care if the bank defaulted with a net liability of one dollar or one billion dollar. The society that has to cope with the fallout (maybe a bailout) should care.

The most prominent issue for internal coherence might be the fact that a risk measure has only one minimum. Otherwise behaviour is erratic and has nothing to do with risk. For example, we would expect that if a portfolio is diversified more and more that the risk decreases more and more. Mathematicians would say that the risk measure needs to be "convex". This property ensures that we only find one optimum. If we would allow for local optima this means that our risk measure is internally incoherent and cannot be used to optimize portfolios. Such metric would be worthless as risk measure and does not deserve the title risk measure.

2 The Consequences of Thinking Incoherently

Failing to select a coherent risk measure can have many different consequences, depending on which axiom is breached. For example using an incoherent risk measure as risk limit can encourage extreme risk taking, counteract diversification, be utterly senseless, create blindness for investment costs, etc.

In this short paper we will only illustrate the two first extremely important aspects (extreme risk taking and diversification) for one risk measure Value at Risk (henceforth VaR). This risk measure is an important one, because since the Basel II agreements VaR is to be used as risk limit on banks. Also UCITS IV enforces VaR as a risk limit for all investment funds that fall under the regulation.

VaR is defined as the smallest(!) loss that one can expect given a certain confidence level. One will appreciate that knowing the smallest loss that one can expect in the 1% worst cases is not so useful. Would the average or maximum loss not be more relevant as a risk measure? The answer to that question is of course "yes". That is our intuition speaking here, let's now illustrate the wisdom of our intuition with an example.

Assume that a bank has a choice of totally independent bonds (different countries, industries, etc.) to buy for its own portfolio, that all have a really small probability to default (assume 0.07% default probability), and assume that if they default they pay nothing. If the bond does not default (so in 99.93% of all cases), it will pay in one year 100% capital back plus 5% interest. Assume that this bank has a 1% VaR limit of 20%.

The smart banker wants to diversify over two independent bonds, and calculates his VaR: it is 47.5%, he tries to diversify more and more in order to decrease the VaR and he finds that he can satisfy the risk limit with five bonds. The smart banker is happy and the regulators also.

Now assume a smarter banker, who calculates the VaR of just one bond (so a totally un-diversified position!). The smarter banker finds that his VaR is 0%! The smarter banker is very happy and the regulator sneers to the smart banker that he should take an example to the smarter banker. The smarter banker is honoured as an example of risk management and he is able to take on loads of other risks (he still has 20% of his VaR limit available!).

The intelligent reader will see the similarities with Dexia's 2011 story and probably find it a devastating finding. But there are more examples: replace now "bond" by "real estate project" and one recognizes the 2008 story of Lehman Brothers.

3 Conclusion

Using VaR as a risk limit is just one way of failing to think coherently about risk. VaR is not convex and hence can never in any set of axioms be coherent. Using VaR as a risk limit encourages extreme risk taking, and on top of that can counter-act diversification. The example above illustrates what our intuition told us: VaR is incoherent as a risk measure, because it

1. can counteract diversification: one bond is considered as less risky than two independent bonds by VaR;

2. is not convex (i.e. it can show false local minima): for example one bond is a false minimum, but careful calculations show that there are also false minima at 22, 64, etc. bonds;

3. can encourage extreme risk taking: a 0.7% probability of a 100% loss is considered as much less risky by VaR than a 0.0049% probability to loose 100% that is accompanied by a 1.4% probability to loose only 50%. Another—much more worrying—example could be that a 1% VaR does not see *any* difference between a 0.07% probability to loose 1% or to loose 100% (provided that these are the smallest losses associated with that investment) ... of course the market does see such difference and the bank will find a much higher expected (or "hoped for") return for the more risky investment ... and isn't that the story of all banks that needed a bailout because they loaded their balances with CDOs, subprime, and junk bonds?

I believe that the above example illustrates that VaR is not only incoherent, but also very dangerous. It encourages extreme risk taking, and is a sure road to disaster. If we fail to address this issue, then we build a financial system that has meltdowns built in. No finger-pointing to bankers, no overload of regulation, no amount of information and compliance requirements will be able to prevent future disasters to happen if we do not use this information coherently. Using VaR as a risk limit for banks is similar to giving a loaded gun to a child: it is irresponsible beyond the forgiveable.

If we fail to use a coherent risk measure (such as the average of losses below the VaR), then we prepare disaster for our children and if we don't learn from the Global Meltdown and the European Debt Crisis, then we might live to see the results. But, beware, the reality is even more complex than that. Risk is inherently endogenous to the financial system. This means that even a coherent risk measure will not completely safeguard us, although it would do a better job.

4 Postscripti

1. This is the first of a series of popular papers on the subject. All knowledge and much more is also in my academic paper "Thinking Coherently", and the papers referenced there, but these papers are much less readable for the non-mathematically trained reader. For a university level introduction to the mathematics behind it, I recommend my book "Maslowian Portfolio Theory".

2. Claiming that thinking incoherently is the recipe for disaster is of course easy after the facts, but many authors have actually with smashing accuracy predicted the Global Meltdown: see for example Danielsson et al. 2001.

3. While this paper is written with Basel II (and Basel III for all that matters) in mind, its findings hold also for UCITS IV or any other regulation that tries to use VaR as a risk limit.

4. Note that VaR is coherent for elliptic distributions (for example the normal or Gaussian distribution is an elliptic distribution). However financial markets are characterized by much more extreme risks than the Gaussian distribution would want us to believe. Even if all assets would be follow a Gaussian distributed, even then each investor has many very non-Gaussian products available (for example all structured products).

5. A coherent risk measure will produce results that are in line with our intuition, e.g. Expected Shortfall (this is the average of all losses worse than VaR) would produce in our example for one bond Expected Shortfall is 70.0%, and for two bonds it is 47.5%. A coherent risk measure can not be fooled, however it is only a view on risks, not on rewards. [for the mathematicians: this means that one coherent risk measure is not a

sufficient condition for second order stochastic dominance of the second order and hence not a sufficient condition for congruence with expected utility theory]. Also a coherent risk measure is not and cannot be a "risk-and-reward indicator" (as the implementation measures of UCITS IV try to do and that even in a very incoherent way ... but that is for the next paper)